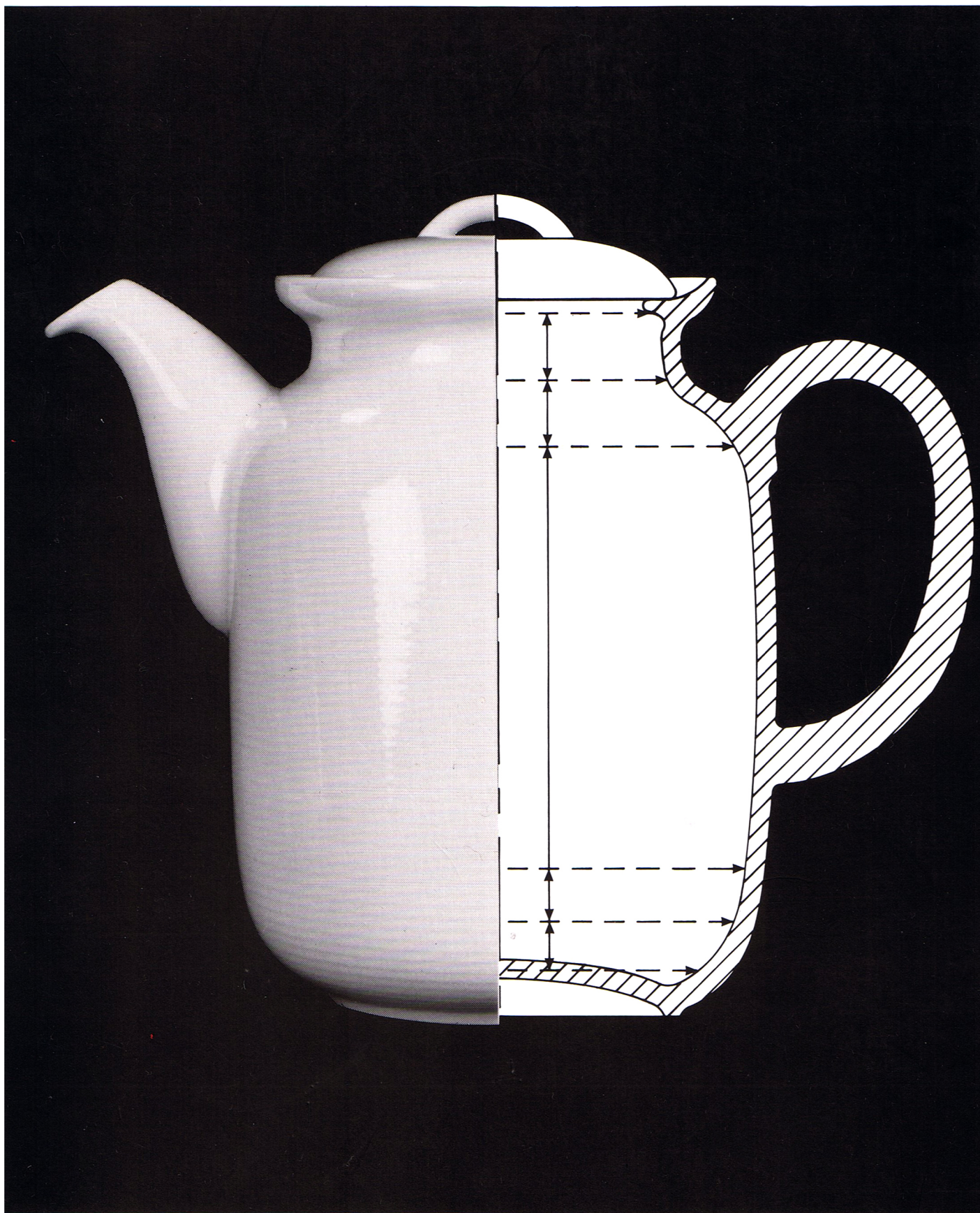


VOLUMETRIC CALCULATIONS

FOR DESIGNERS & CRAFTSPEOPLE



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PUBLISHED BY POTTERYCRAFTS LTD

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Acknowledgements

In the preparation of this work we would like to acknowledge the help and support we have received from the following:

Wedgwood Group
North Staffs Polytechnic
John Lewis Partnership
Dr. Moulton
Professor M. French
United Glass
Pottery crafts Limited

Introduction

Designers, craftsmen and modellers need to be able to determine with accuracy the volume of a vessel. If it exists as a prototype, this can be done easily by filling it with water from a graduated cylinder or by weighing the water that is needed to fill it. If the design, however, exists only as a drawing or solid model, other methods have to be used. Design usually starts on paper, but not always, and it is impossible when designing let us say a wine glass, carafe or teapot to be sure that what you have put on paper will have the required capacity. It is important to be able to determine accurately the volume of what you have drawn and then if necessary adjust the drawing to give a required capacity.

Most people with a modest knowledge of maths know how to calculate the volume of a cube, cylinder or sphere, but the objects we are concerned with do not usually fall into these categories. There are no simple mathematical formulae to calculate the volume of these solids. In order to calculate the volume, it is necessary to divide them up into component parts such as cones or cylinders or break them down into slices in the way that salami is cut in a delicatessen. It is possible to calculate the volume of any 3-dimensional object with reasonable accuracy by establishing the sectional area of parallel slices at regular intervals. The more slices the greater the accuracy. This is, of course, easy to do if the object has a circular cross-section as the cross-sectional area of any slice can be derived from its radius. Unfortunately, many practitioners of design and modellers working in the ceramic industry have been taught to do this incorrectly. The system that is often used is to measure the sectional radii, average these and calculate the volume as a cylinder. In certain cases this method can give an error of such magnitude that it renders the calculation invalid. Using the same measurements, it is possible to perform an accurate volumetric calculations by employing a slightly different technique called Durand's Method.

In the following chapters, various techniques of volume calculation are given: these include the universally applicable Durand's Method, and some quicker methods based on geometric shapes such as cones and cylinders, and an intriguing method known as Pappus' Theorem.

The subject of volumetric calculations has not been covered previously for people with minimal mathematical knowledge. The ways that are recommended for these calculations are in many cases approximations, but with care, should give results accurate to a few percent. There are some tricks of the trade that are useful, like how to model a figurative piece, such as a Toby Jug, at a predetermined capacity. Few mathematicians would have any idea how this could be done.

DAVID QUEENSBERRY

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1. Volume Calculations of Solid Objects

It is normally recommended that brimful capacity should be measured. In the case of vessels with an internal fitting lid, the capacity should be measured to the underneath of the verge that supports the lid. Some coffee and tea pots overflow before the liquid rises to this point and if this is the case the capacity should be measured to this height.

With containers for liquids such as beer, wine and spirits, capacity has to take into account headspace: this is the capacity in the container above the point to which it will be filled. The head space is expressed as a percentage of the volume of the liquid that will be put in the container. This is termed the vacuity.

With a normal wine bottle holding 750cc and with a cork closure, the vacuity will be between 3.5 and 4.5%, giving a bottle with a brimful capacity of 776cc to 783cc.

MEASURING CYLINDERS

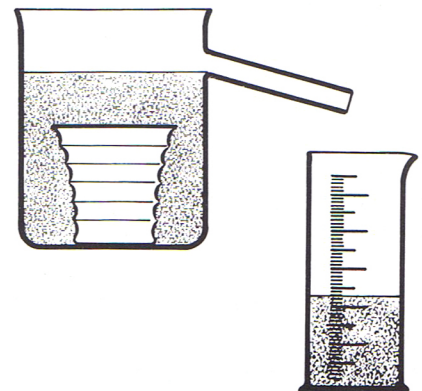
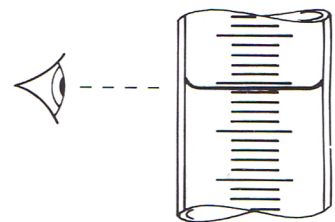
Fill the measuring cylinder accurately to a capacity greater than the volume of the vessel that you want to measure (volume A). Pour water from the cylinder into the vessel and then note the volume of the water left in the flask (volume B). The volume of the vessel is A-B. Always use the smallest diameter cylinder possible.

Taking the reading of a measuring cylinder is confused by the water climbing up the sides around the contact points with the glass, an action caused by surface tension. For accuracy the reading must be taken to the general level of the water away from the sides.

DISPLACEMENT

Any solid object that is immersed in a liquid displaces its own volume. This simple principle is evidenced when you get in the bath and the water rises. Archimedes was the first person to see the scientific importance of this. It is possible to determine the volume of a solid object by lowering it into a tank full up with water and measuring the volume of the water that overflows. Obviously, this would be a messy system so it is necessary to have a tank with an overflow pipe. The tank is then filled to the level of the overflow pipe. The object in question is lowered into the tank, causing water to be displaced through the overflow pipe. The volume of the displaced water is the same as that of the immersed object.

It needs to be noted that objects denser than water will sink by their own weight but objects that float need to be pushed under using a stiff, thin rod or immersed by attaching weights whose displacement can be determined separately. The other problem that may be encountered is porosity. If the object absorbs water the result will be inaccurate. This can be a problem with plaster, which is the most commonly used material for ceramic model making. The answer is to make sure the plaster model has been soaked thoroughly, or coated with a material that renders it impervious.



Weight

It is now possible to buy for a modest price accurate electronic kitchen scales with a digital display. The density of water is 1.00 (1g = 1cc) so any weight of water in grammes can be transposed into a volumetric measurement in ccs. It is far easier to check volume by weight using electronic scales than by using a measuring cylinder. First weigh the vessel whose volume you wish to establish, then fill it with water and weigh again. The volume is the difference between the two weights, grammes equalling ccs. If you weigh in ounces then you have immediately got the capacity in fluid ounces. Most kitchen scales have a zero key so that you can cancel the weight of the vessel before filling it with water so it is only necessary to weigh once.

Densities

The density of an object is defined as the weight of the object divided by its volume. The density of water is 1.00g/cc whereas the density of fired ceramic material is approximately 2.5g/cc (ie. every cubic centimetre weighs 2.5g).

The volume of materials that make up an object can be calculated if the density and weight of the material is known. For example, an empty teacup has a weight of 225g. The volume of the ceramic material used can be calculated by dividing the weight of the teacup by the density of the constituent ceramic (2.5g/cc). Thus the volume of the ceramic material used in the construction is $225 \div 2.5$ or 90cc. This information is particularly useful when determining volumes by complete displacement. The internal volume of an object equals the displacement volume less the material volume. A table of densities of common materials is given in Section 7.

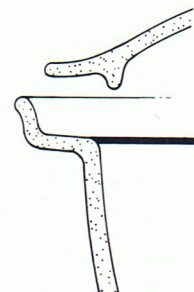
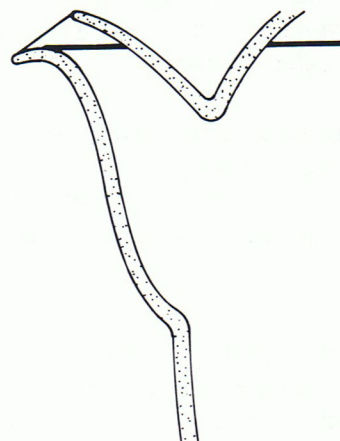
Snips and Spouts

Jugs, coffeepots and teapots have some capacity in their pouring appendages, normally volumetric calculations disregard this. When making volumetric calculations of these vessels the following capacity increases are suggested:

Pots with short spouts	65 mm	add 20 cc
Pots with long spouts	90 mm	add 30 cc
Cream jugs		add 5 cc
Gravy boats		add 8 cc

Special Case (Toby Jug Method)

It is possible to model an irregular ceramic form like a Toby Jug to a given capacity without making a calculation. There is a reasonably accurate relationship between the volume of a clay sized model, and the fired size vessel produced from the model, provided the forms are of a general similarity. If it is known that a one pint Toby Jug is modelled from a particular volume of clay, then other Toby Jugs modelled from the same volume of clay will also have a one pint capacity. This system works if you assume the same ceramic body, firing temperature and wall thickness.



2. Volume Calculations of Designed Objects

For vessels at the design stage, prior to three-dimensional modelling, the methods demonstrated in the following sections can be used to calculate volumes. Whilst in most practical cases, it is usual to make certain approximations (decreasing the overall calculation time), it has been found that the methods described yield both reasonably accurate and useful results.

There are four basic methods of calculation:

- 1) Use of mathematical equations that describe or approximate closely the form of a vessel-drawing **a** & **b** on facing page.
- 2) Dividing the solid in question into a series of convenient conical or pyramidal sections that closely approximate the overall shape, calculating the volume of each section by simple formulae, and adding the volumes of each section together – Drawing **c**.
- 3) Dividing the solid into small slices of equal height and applying Durand's rule to the dimensions of each section to obtain the volume, the more slices the more accurate the result, (called Durand's method) – drawing **d**.
- 4) Pappus' Theorum – drawing **e**.

Dividing an object up into many, very thin slices with the subsequent calculations is a task best suited to a digital computer. Computers are used for this purpose where very high accuracy and speed are required. It is anticipated that, with the advent of computer-aided design within the industry, more and more calculations will be performed using computers.

All the methods described here use simple arithmetic and are best performed using the provided calculation sheets with the aid of a pocket calculator.

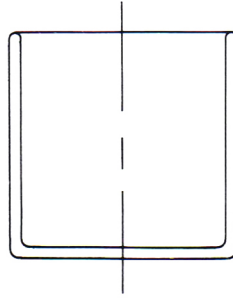
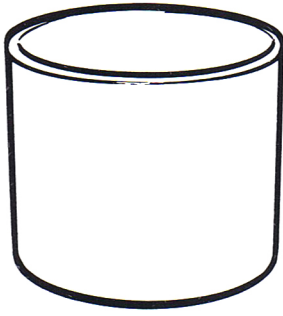
Standardised notation is used throughout all the diagrams, examples and calculations as described below.

MAJOR RADII (radii perpendicular to a central axis)	$R_1 R_2$	ELLIPTICAL MAJOR AXIS	$a_1 a_2$
		ELLIPTICAL MINOR AXIS	$b_1 b_2$
OTHER RADII	$r_1 r_2$	AREA	$A_1 A_2$
HEIGHT	$h_1 h_2$	OTHER VARIABLES	$D N W$
LENGTH	$x_1 x_2$	CONSTANT π	(3.1416)

It is strongly recommended that each calculation is performed with a layout identical to the ones given in the following examples. Blank calculation sheets are provided for this purpose towards the rear of this book.

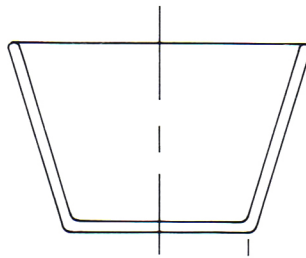
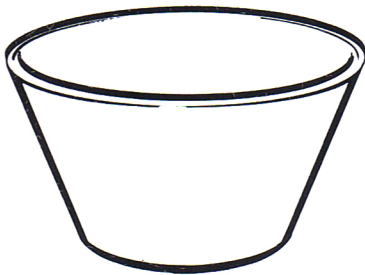
All measurements are worked throughout in millimetres. The resultant is divided by 1,000,000, converting cubic millimetres (millilitres) into litres, giving a figure for the volume which is easier to manage and understand.

2.1 Round Forms–Index of Calculating Methods



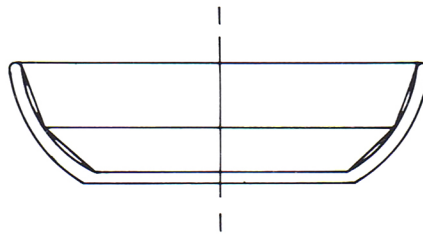
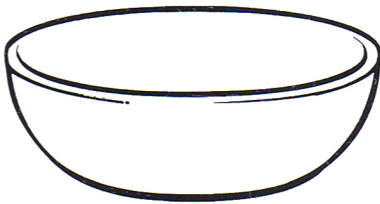
a. Cylindrical Forms **Page 8**

Use of simple formula using two measurements to obtain volume.



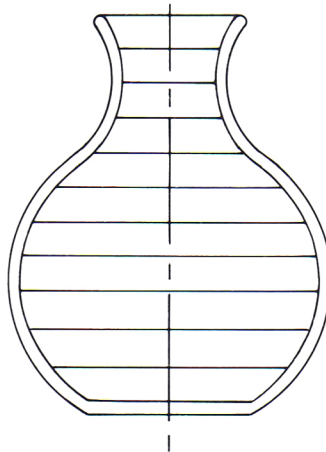
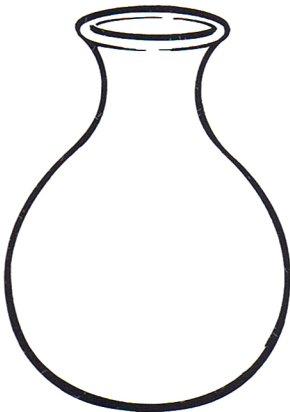
b. Truncated Cones **Page 9**

Use of simple formula using three measurements to obtain volume.



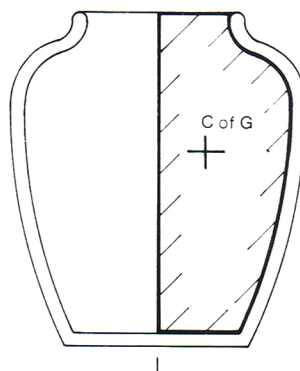
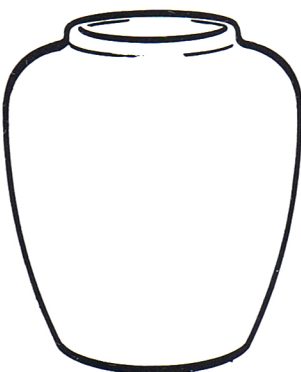
c. Addition of Cones **Page 10**

Close approximation of form by dividing into a number of cones. Volume calculation achieved by summation of simple formula.



d. Durand's Rule **Page 12**

Dividing form into equal intervals and applying Durand's rule to obtain volume from dimensions of each interval.

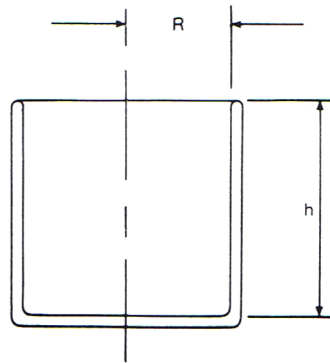


e. Pappus' Theorem **Page 14**

Cutting half the profile out in heavy paper, determining both the area and the centre of gravity of the cut shape and using a simple formula to obtain volume.

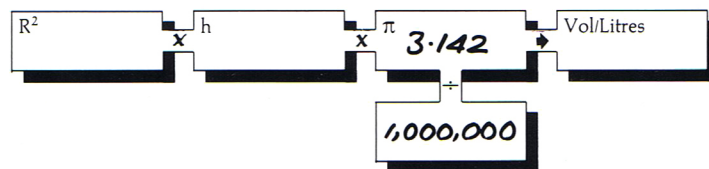
Cylindrical Forms

Volume = $\pi R^2 h$

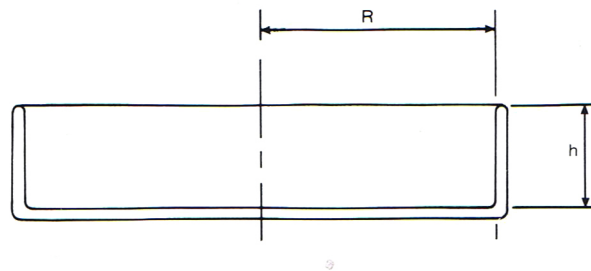


Radius R = _____

Height h = _____

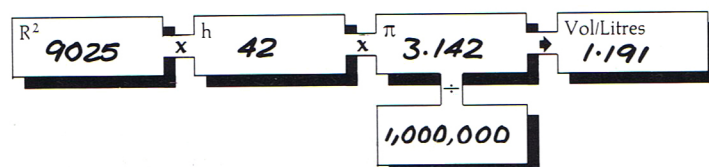


Example



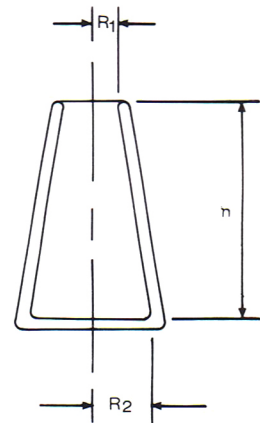
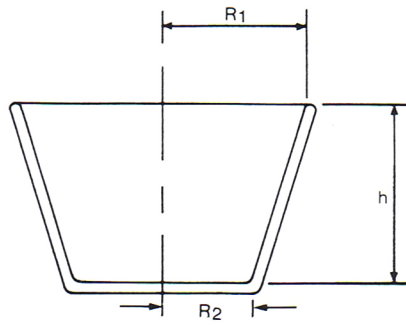
Radius R = 95

Height h = 42



Truncated Forms

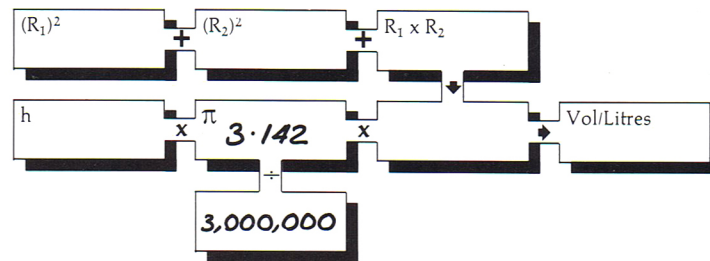
$$\text{Volume} = \frac{\pi}{3} h [R_1^2 + R_2^2 + R_1 R_2]$$



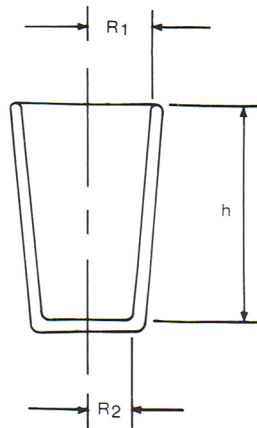
Top Radius $R_1 =$ _____

Height $h =$ _____

Bottom Radius $R_2 =$ _____



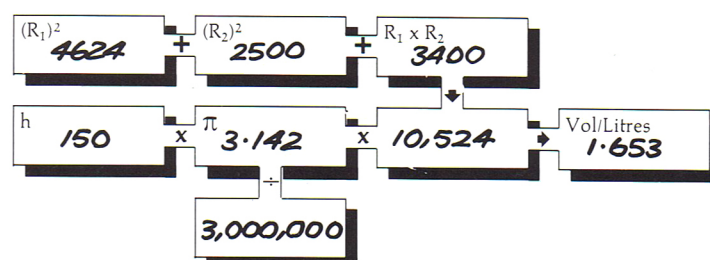
Example



Top Radius $R_1 =$ 68

Height $h =$ 150

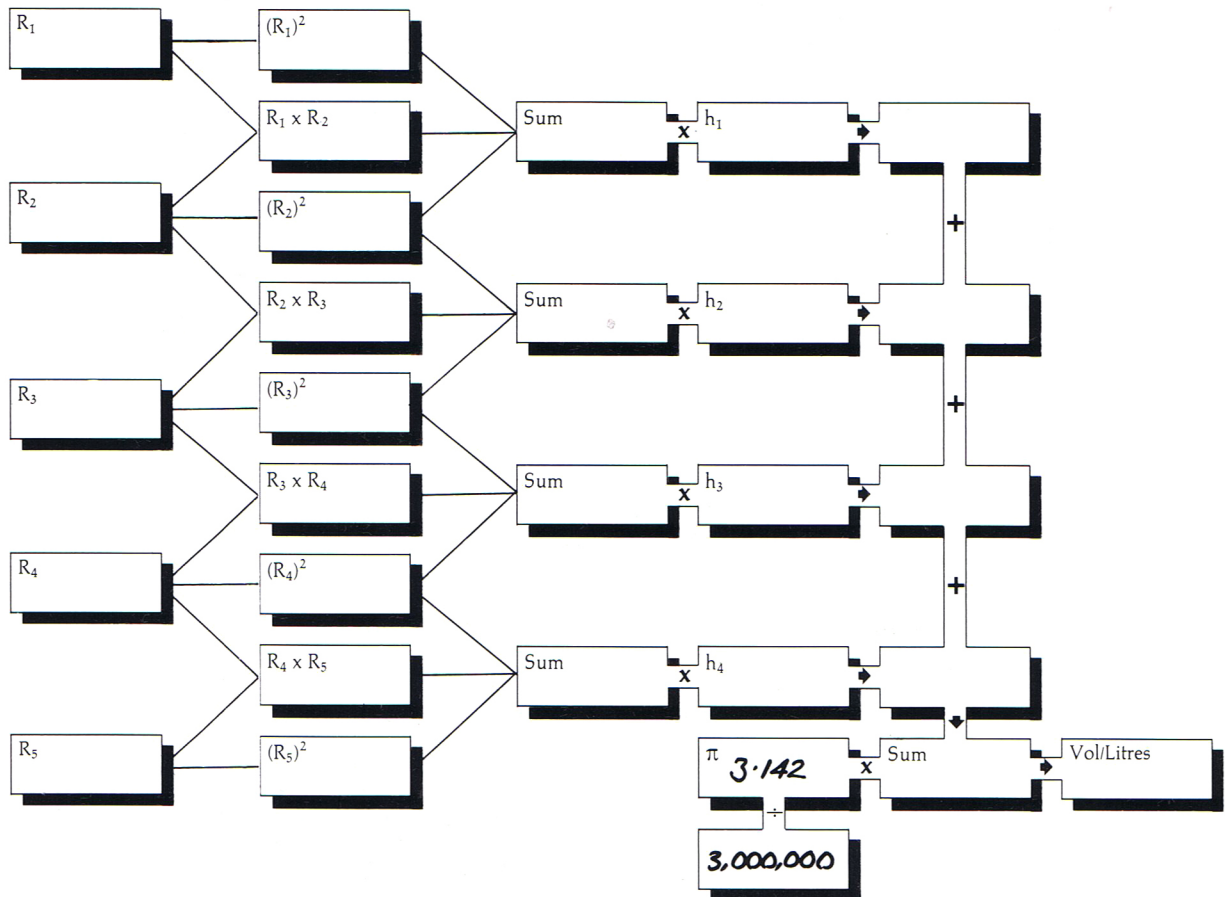
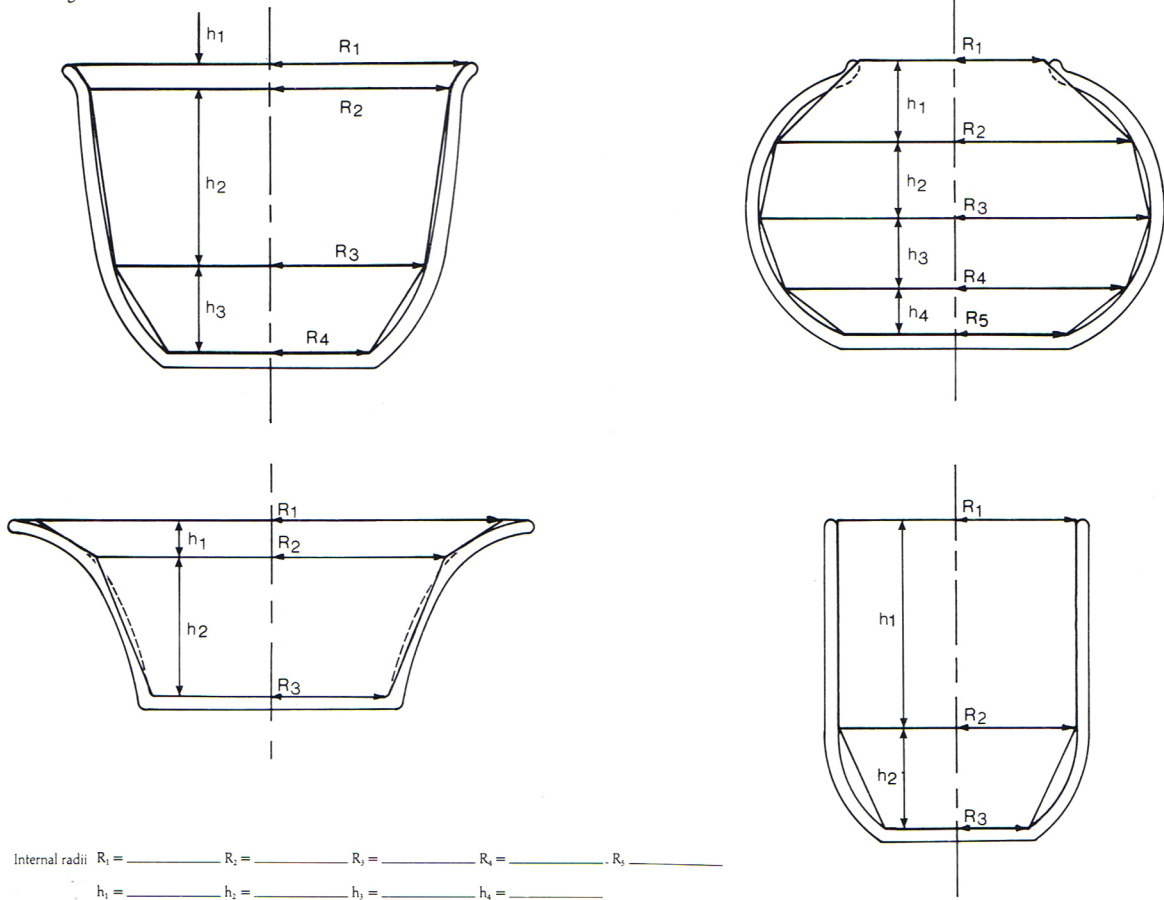
Bottom Radius $R_2 =$ 50



Addition of Cones

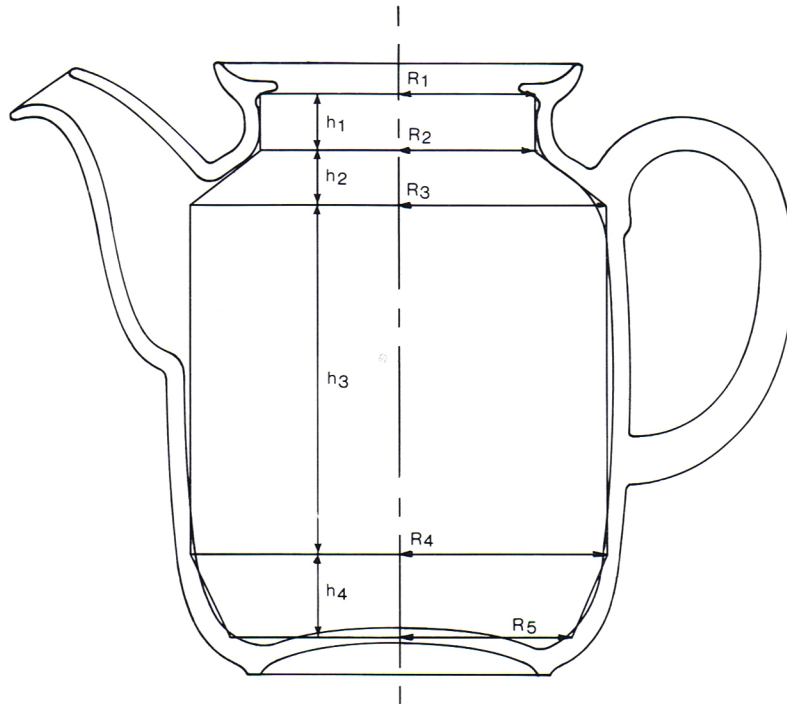
2, 3, & 4 Truncated Cone Combinations

$$\text{Volume} = \frac{\pi}{3} [h_1(R_1^2 + R_2^2 + R_1 R_2) + h_2(R_2^2 + R_3^2 + R_2 R_3) + h_3(R_3^2 + R_4^2 + R_3 R_4) + h_4(R_4^2 + R_5^2 + R_4 R_5)]$$



Example, Thomas Coffee Pot (as shown on front cover)

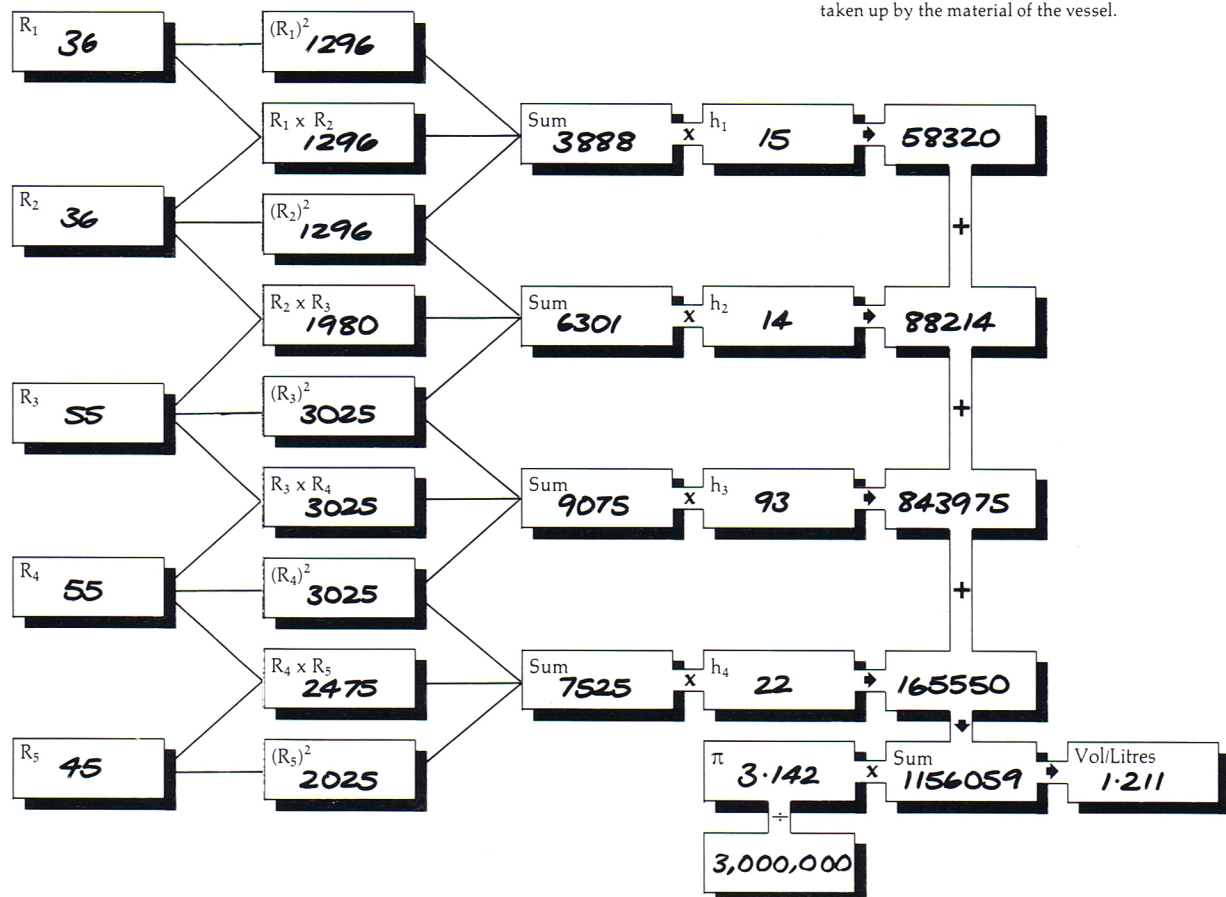
Calculation by Cones



Internal radii $R_1 = \frac{36}{15}$ $R_2 = \frac{36}{14}$ $R_3 = \frac{55}{93}$ $R_4 = \frac{55}{22}$ $R_5 = \frac{45}{22}$

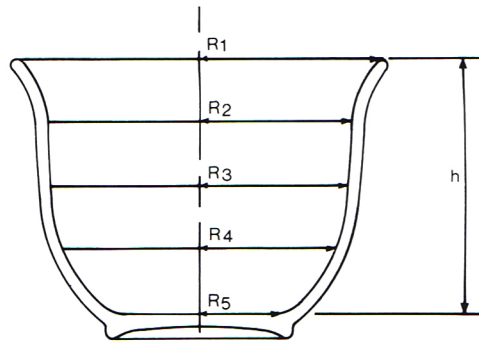
When the internal volume is divided into cones an inaccuracy appears as the conic sections can only approximate the internal curved form.

To increase the accuracy of the calculations the radius measurements of the conic sections can extend a little into the body making the straight line of the conic section average out the measured volume space between the internal space and that taken up by the material of the vessel.

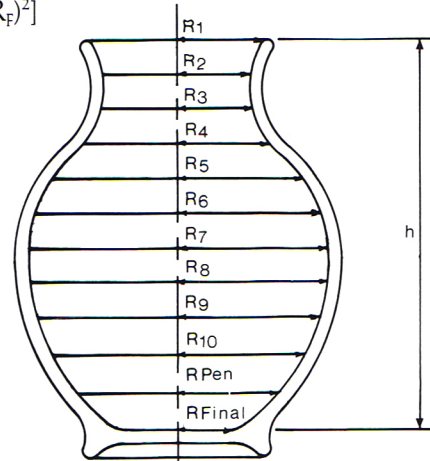


Durand's Rule (Circular)

$$\text{Volume} = \frac{\pi h}{N} [(0.4 (R_1)^2 + 1.1 (R_2)^2 + (R_3)^2 + (R_4)^2 + \dots + 1.1 (R_p)^2 + 0.4 (R_f)^2)]$$



Minimum 4 equal intervals
for simpler profiles



Up to 12 equal intervals
for more complex profiles

Radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____ $R_5 =$ _____

$R_6 =$ _____ $R_7 =$ _____ $R_8 =$ _____ $R_9 =$ _____ $R_{10} =$ _____

R (Penultimate) = _____

R (Final) = _____

Overall height h = _____

No. of intervals N = _____

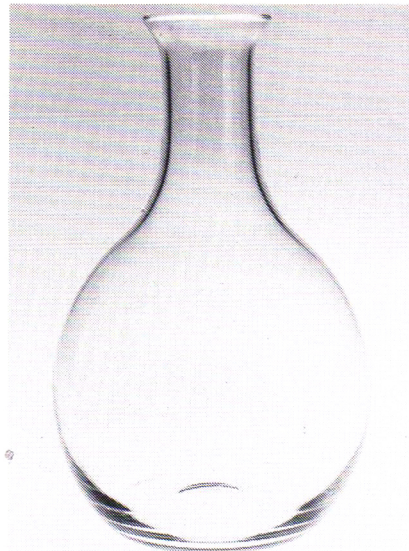
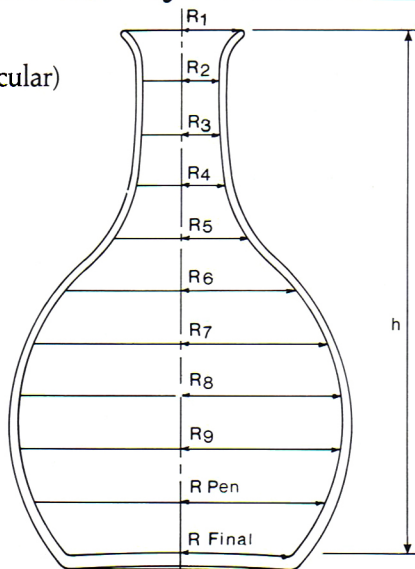
R_1	$(R_1)^2$	x	0.4	+	
R_2	$(R_2)^2$	x	1.1	+	
R_3	$(R_3)^2$			+	
R_4	$(R_4)^2$			+	
R_5	$(R_5)^2$			+	
R_6	$(R_6)^2$			+	
R_7	$(R_7)^2$			+	
R_8	$(R_8)^2$			+	
R_9	$(R_9)^2$			+	
R_{10}	$(R_{10})^2$			+	
R Penultimate	$(R \text{ Penultimate})^2$	x	1.1	+	
R Final	$(R \text{ Final})^2$	x	0.4	+	
h		x	π		Sum
N			3.142		
			1,000,000		Vol/Litres

The number of radii inserted is dependent on the number of intervals chosen.

The penultimate and final radii are always inserted at the end.

Example, Reijmyre Glass Decanter

Calculation by
Durand's Rule (Circular)

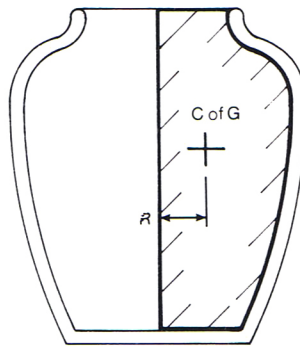


Radii $R_1 = 26$ $R_2 = 16$ $R_3 = 17$ $R_4 = 19$ $R_5 = 30$
 $R_6 = 50$ $R_7 = 64$ $R_8 = 70$ $R_9 = 70$ $R_{10} =$
 R (Penultimate) = 63 R (Final) = 49
 Overall height $h = 232$ No. of intervals $N = 10$

R_1	26	$(R_1)^2$	676	x	0.4	270.4	
R_2	16	$(R_2)^2$	256	x	1.1	281.6	+
R_3	17	$(R_3)^2$	289				+
R_4	19	$(R_4)^2$	361				+
R_5	30	$(R_5)^2$	900				+
R_6	50	$(R_6)^2$	2500				+
R_7	64	$(R_7)^2$	4096				+
R_8	70	$(R_8)^2$	4900				+
R_9	70	$(R_9)^2$	4900				+
R_{10}		$(R_{10})^2$					+
R Penultimate	63	$(R \text{ Penultimate})^2$	3969	x	1.1	4365.9	+
R Final	49	$(R \text{ Final})^2$	2401	x	0.4	960.4	+
h	232	23.2		x	π 3.142	Sum 23824.3	Vol/Litres
N	10				1,000,000		1.737

Pappus' Theorem

$$\text{Volume} = 2 \pi AR$$



By locating the Centre of Gravity (C of G) and determining the area of half the internal vertical section Pappus' Theorem can be applied using the above formula.

The Centre of Gravity (C of G) is found by cutting the half profile out of card, and suspending it from at least two points around the periphery allowing the cut-out to fall freely, and marking on the cut-out where the perfect vertical lies for each suspension point. The place at which the vertical lines cross indicates the C of G. The radius from the central axis of the object to the C of G is found by direct measurement.

The area is found either by calculation using standard formulae for areas listed on page 45, by weighing or more easily by square counting.

Square counting requires the half profile to be cut out, or marked out on millimetre graph paper. The area is given directly by the addition of all the squares within the shape.

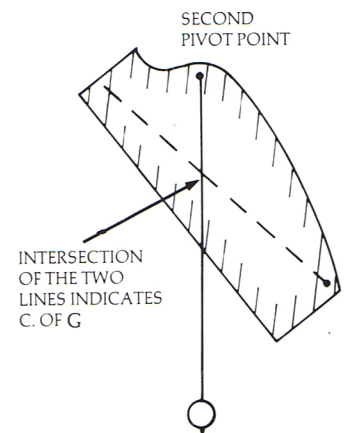
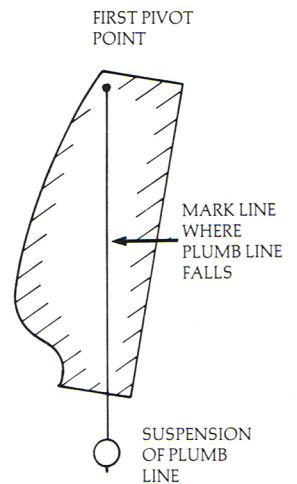
Area of half profile $A = \underline{\hspace{2cm}} \text{ mm}^2$
 Radius of C of G (millimetres) $R = \underline{\hspace{2cm}} \text{ mm}$

A	x	R	x	π	3.142	Vol/Litres
500,000						

The weighing method requires the use of an accurate balance to determine the weight of half the profile shape and also the weight of a square, measuring 100mm x 100mm, cut from the same piece of card.

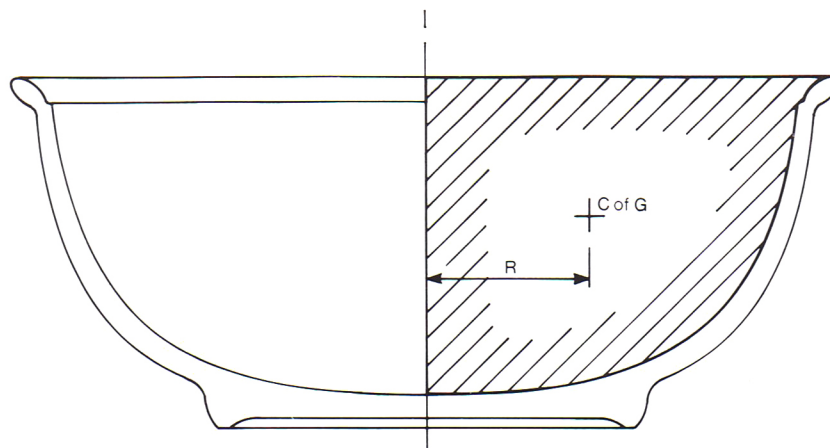
Weight of profile (grammes) $W_p = \underline{\hspace{2cm}} \text{ g}$
 Weight of square (grammes) $W_s = \underline{\hspace{2cm}} \text{ g}$
 Radius of C of G (millimetres) $R = \underline{\hspace{2cm}} \text{ mm}$

W_p	x	R	x	$\pi \div 50$	0.0628	Vol/Litres
W_s						



Example, Thomas Bowl

Calculation by Pappus' Theorem (Circular)

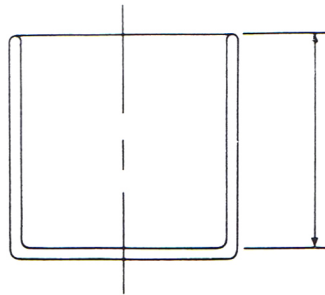
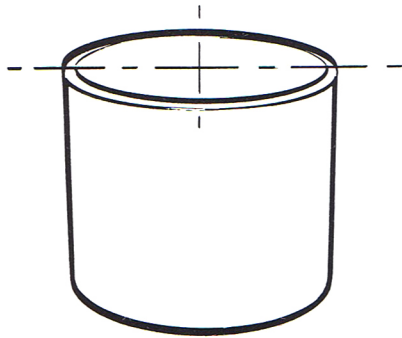


Area of half profile $A = \underline{7093} \text{ mm}^2$

Radius of C & G $R = \underline{43} \text{ mm}$

A	7093	x	R	43	x	π	3.142	→	Vol/Litres	1.917
									+	500,000

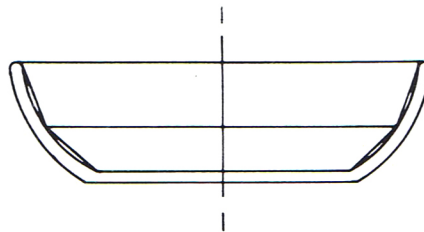
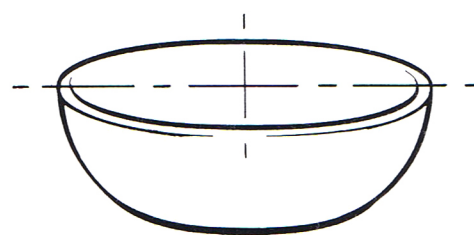
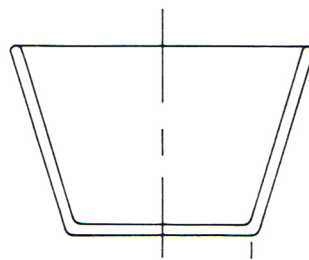
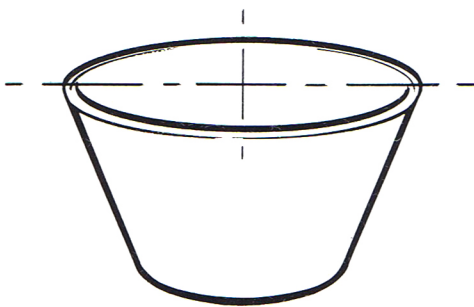
2.2 Elliptical Objects- Index of Calculating Methods



a. Straight Sided

Page 17

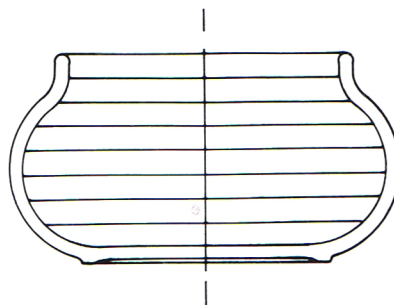
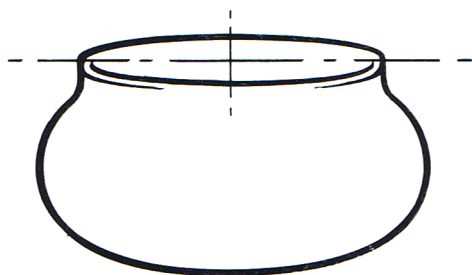
Use of simple formula using five measurements to obtain volume.



b. Addition of Cones

Page 18

Close approximation of form by dividing into a number of cones. Volume calculation achieved by summation of simple formula.



c. Durand's Rule

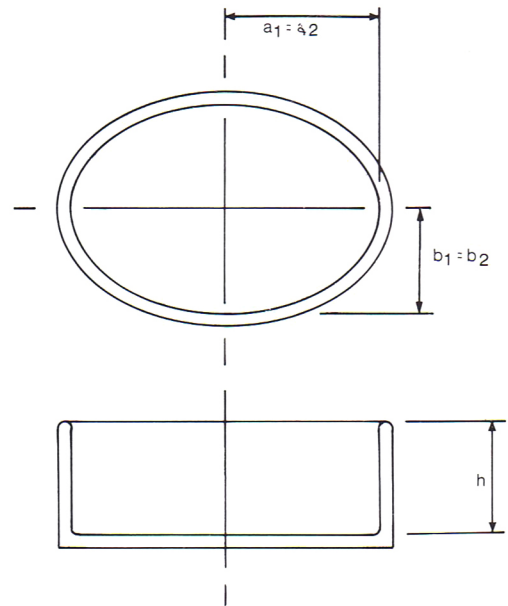
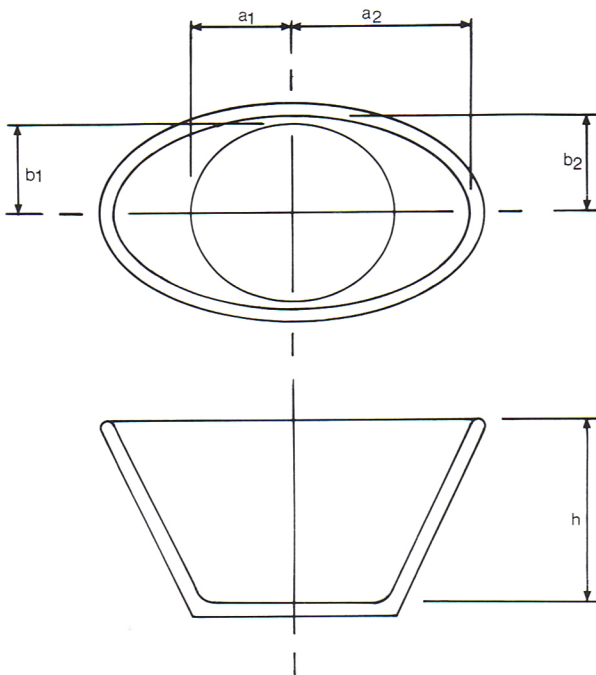
Page 20

Dividing form into equal intervals and applying Durand's rule to obtain volume from dimensions of each interval.

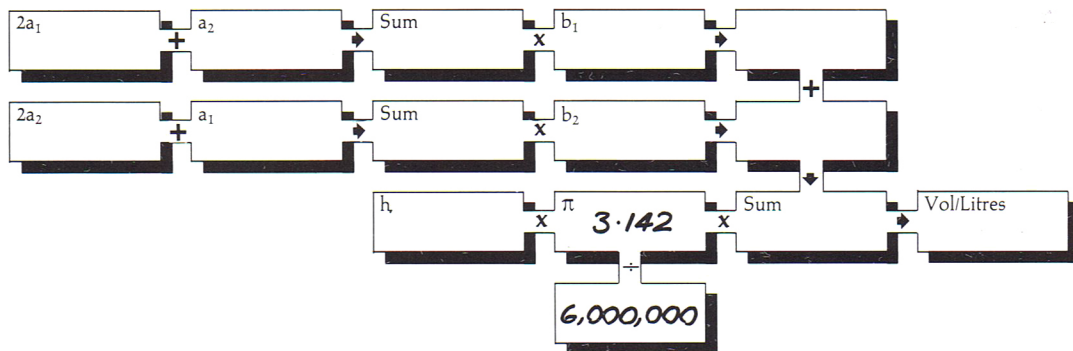
Straight Sided Elliptical Forms

$$\text{Volume} = \frac{\pi h}{6} [(2a_1 + a_2)b_1 + (a_1 + 2a_2)b_2]$$

Different shape ellipses possible top & bottom

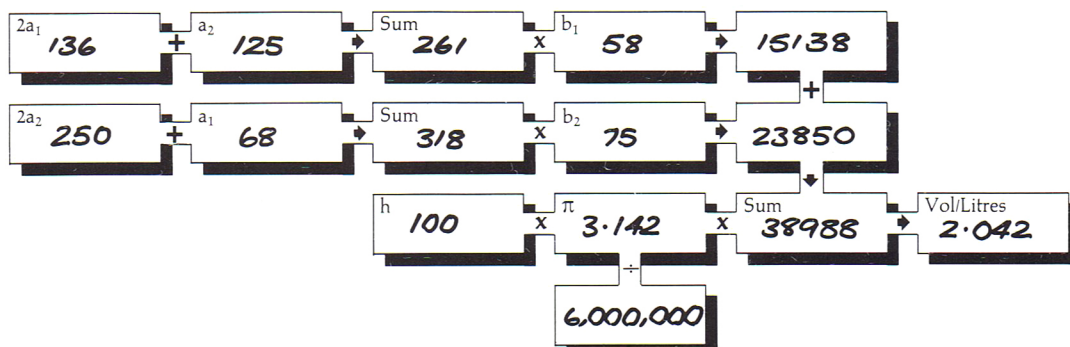


Major axis radii $a_1 =$ _____ $a_2 =$ _____
 Minor axis radii $b_1 =$ _____ $b_2 =$ _____
 Height $h =$ _____



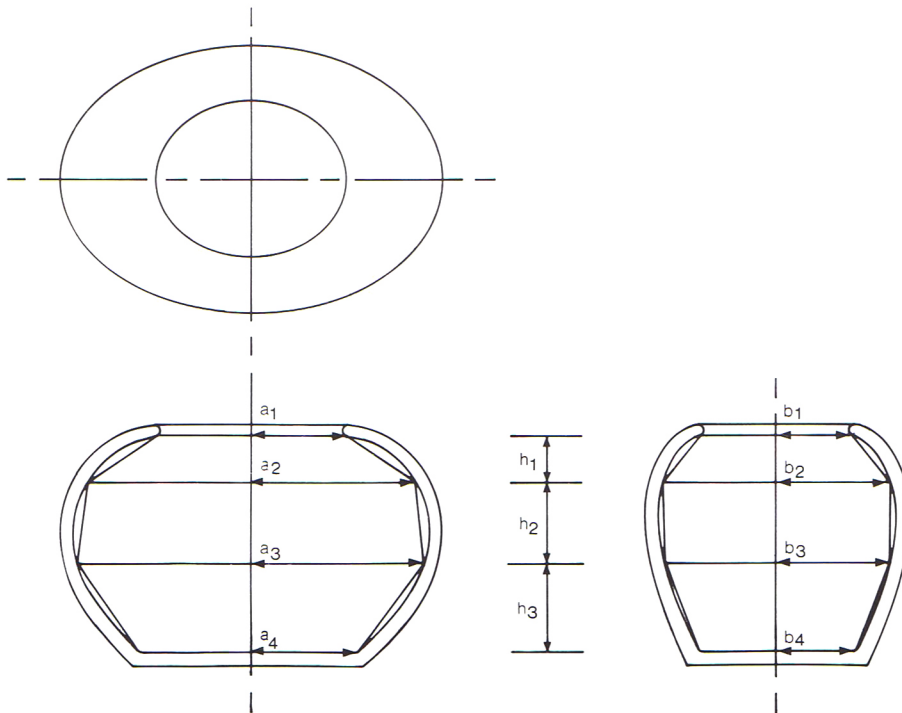
Example

Major axis radii $a_1 = 68$ $a_2 = 125$
 Minor axis radii $b_1 = 58$ $b_2 = 75$
 Height $h = 100$



Addition of Cones (Elliptical)

$$\text{Volume} = \frac{\pi}{6} [(2a_1+a_2)b_1h_1+(a_1+2a_2)b_2h_1+(2a_2+a_3)b_2h_2+(a_2+2a_3)b_3h_2+(2a_3+a_4)b_3h_3+(a_3+2a_4)b_4h_3+(2a_4+a_5)b_4h_4+(a_4+2a_5)b_5h_4]$$

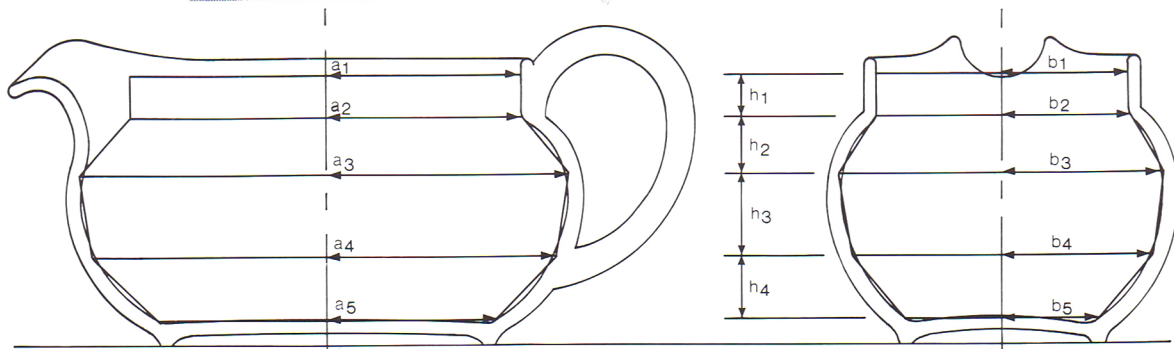
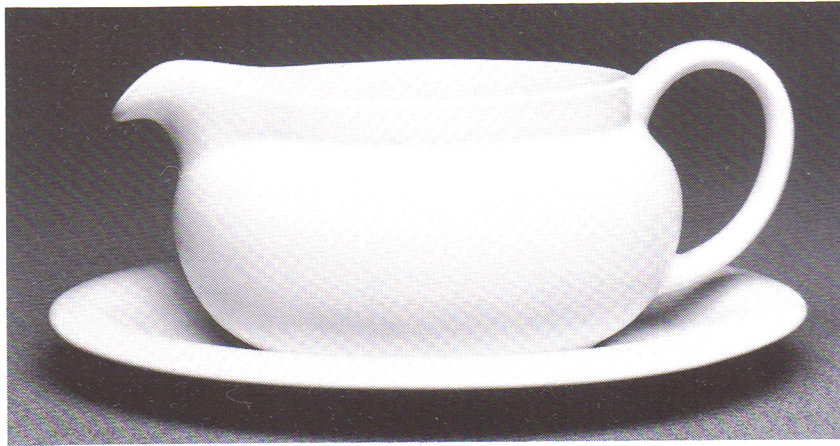


Height $h_1 =$ _____ $h_2 =$ _____ $h_3 =$ _____
 $a_1 =$ _____ $a_2 =$ _____ $a_3 =$ _____
 $b_1 =$ _____ $b_2 =$ _____ $b_3 =$ _____
 $b_4 =$ _____ $b_5 =$ _____

a_1	$2a_1$	$2a_1+a_2$	\times	b_1	\times	h_1	$+$
a_2	$2a_2$	a_1+2a_2	\times	b_2	\times	h_1	$+$
a_3	$2a_3$	$2a_2+a_3$	\times	b_2	\times	h_2	$+$
a_4	$2a_4$	a_2+2a_3	\times	b_3	\times	h_2	$+$
a_5	$2a_5$	$2a_3+a_4$	\times	b_3	\times	h_3	$+$
		a_3+2a_4	\times	b_4	\times	h_3	$+$
		$2a_4+a_5$	\times	b_4	\times	h_4	$+$
		a_4+2a_5	\times	b_5	\times	h_4	$+$
π 3.142 \times Sum							Vol/Litres
6,000,000							

Example, Hornsea Gravy Boat

Calculation by Cones



$$\begin{aligned} \text{Height } h_1 &= 11 & h_2 &= 15.5 & h_3 &= 22 \\ h_4 &= 17 \\ a_1 &= 52 & a_2 &= 52 & a_3 &= 65 \\ a_4 &= 62 & a_5 &= 45 \\ b_1 &= 33.5 & b_2 &= 33.5 & b_3 &= 43 \\ b_4 &= 40 & b_5 &= 25.5 \end{aligned}$$

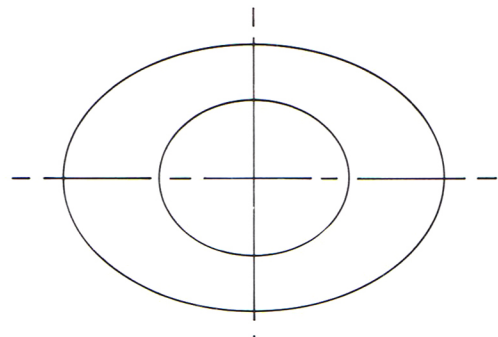
When the internal volume is divided into cones an inaccuracy appears as the conic sections can only approximate the internal curved form.

To increase the accuracy of the calculations the radius measurements of the conic sections can extend a little into the body making the straight line of the conic section average out the measured volume space between the internal space and that taken up by the material of the vessel.

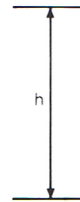
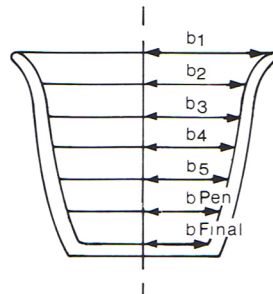
a_1	52	$2a_1$	104	$2a_1 + a_2$	156	\times	b_1	33.5	\times	h_1	11	\rightarrow	57486	+			
a_2	52	$2a_2$	104	$a_1 + 2a_2$	156	\times	b_2	33.5	\times	h_1	11	\rightarrow	57486	+			
a_3	65	$2a_3$	130	$2a_2 + a_3$	169	\times	b_2	33.5	\times	h_2	15.5	\rightarrow	87753	+			
a_4	62	$2a_4$	124	$a_2 + 2a_3$	182	\times	b_3	43	\times	h_2	15.5	\rightarrow	121303	+			
a_5	45	$2a_5$	90	$2a_3 + a_4$	192	\times	b_3	43	\times	h_3	22	\rightarrow	181632	+			
				$a_3 + 2a_4$	189	\times	b_4	40	\times	h_3	22	\rightarrow	166320	+			
				$2a_4 + a_5$	169	\times	b_4	40	\times	h_4	17	\rightarrow	114920	+			
				$a_4 + 2a_5$	152	\times	b_5	25.5	\times	h_4	17	\rightarrow	65892	+			
												\rightarrow	Sum	852792	\rightarrow	Vol/Litres	0.447
													π	3.142	\times		
																	6,000,000

Durand's Rule (Elliptical)

$$\text{Volume} = \frac{h \pi}{N} (0.4a_1b_1 + 1.1a_2b_2 + a_3b_3 + a_4b_4 + \dots + 1.1a_pb_p + 0.4a_fb_f)$$



Ellipses can change from top to bottom

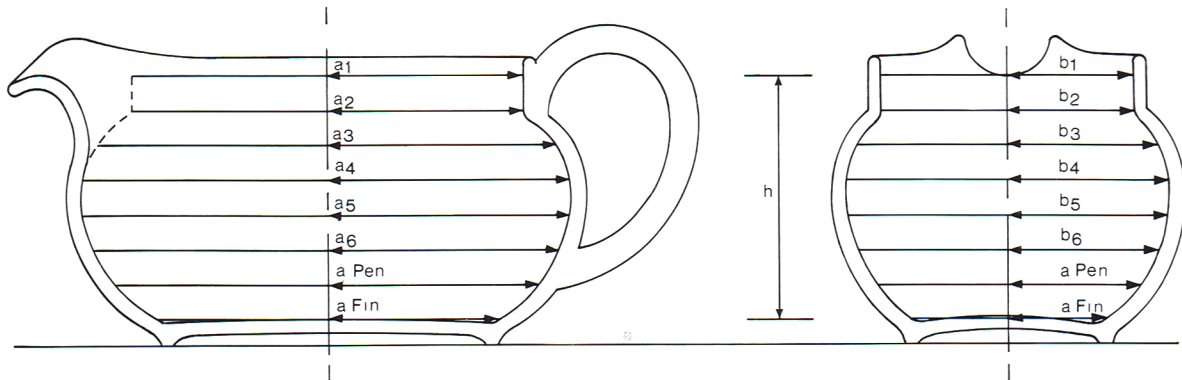


Minor axis $b_1 =$ $b_2 =$ $b_3 =$ $b_4 =$ Major axis $a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$ Height $h =$
 $b_5 =$ $b_6 =$ $b_7 =$ $b_8 =$ $a_5 =$ $a_6 =$ $a_7 =$ $a_8 =$ No of intervals
 $b_9 =$ $b_{10} =$ $b_{\text{Penultimate}} =$ $b_{\text{Final}} =$ $a_9 =$ $a_{10} =$ $a_{\text{Penultimate}} =$ $a_{\text{Final}} =$

a_1	b_1	0.4	
a_2	b_2	1.1	
a_3	b_3		
a_4	b_4		
a_5	b_5		
a_6	b_6		
a_7	b_7		
a_8	b_8		
a_9	b_9		
a_{10}	b_{10}		
$a_{\text{Penultimate}}$	$b_{\text{Penultimate}}$	1.1	
a_{Final}	b_{Final}	0.4	
h	π	3.142	Sum
N		$1,000,000$	Vol/Litres

Example, Hornsea Gravy Boat

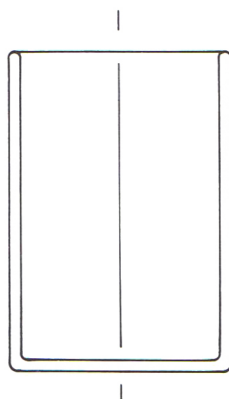
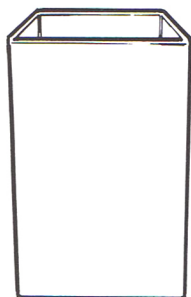
Calculation by Durland's Rule (Elliptical)



Minor axis $b_1 = 33.5$ $b_2 = 33.5$ $b_3 = 39.5$ $b_4 = 42$ Major axis $a_1 = 52$ $a_2 = 52$ $a_3 = 61$ $a_4 = 64$ Height $h = 65$
 $b_5 = 42$ $b_6 = 40$ $b_7 = 35$ $b_8 = 24.5$ $a_5 = 64$ $a_6 = 61.5$ $a_7 = 56$ $a_8 = 44.5$ No of intervals $N = 7$
(Penultimate) (Final) (Penultimate) (Final)

a_1	52	x	b_1	33.5	x	0.4	→	696.8	
a_2	52	x	b_2	33.5	x	1.1	→	1916.2	+
a_3	61	x	b_3	39.5	x		→	2409.5	+
a_4	64	x	b_4	42	x		→	2688	+
a_5	64	x	b_5	42	x		→	2688	+
a_6	61.5	x	b_6	40	x		→	2460	+
a_7		x	b_7		x		→		+
a_8		x	b_8		x		→		+
a_9		x	b_9		x		→		+
a_{10}		x	b_{10}		x		→		+
a Penultimate	56	x	b Penultimate	35	x	1.1	→	2156	+
a Final	44.5	x	b Final	24.5	x	0.4	→	436.1	+
h	65	→	9.286	x	π	3.142	x	Sum	15450.6
N	7							Vol/Litres	0.451
									1,000,000

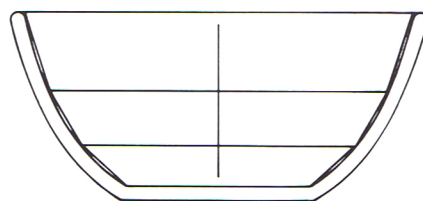
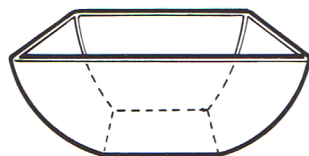
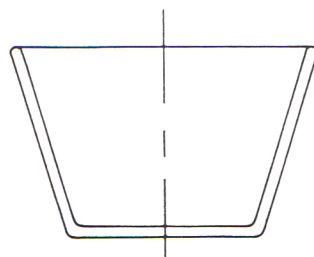
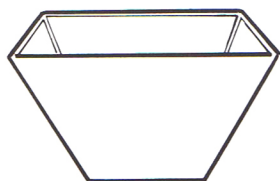
2.3 Geometric Forms Index of Calculating Methods



a. Straight Sided

Page 23

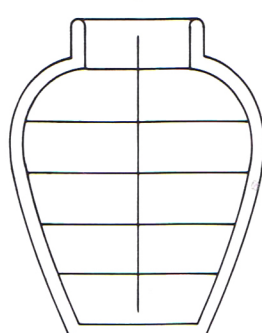
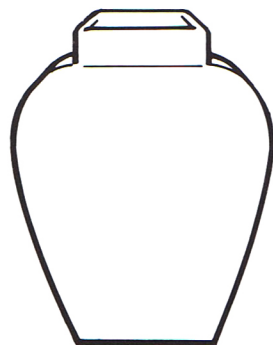
Use of simple formulae using four measurements to obtain volume.



b. Addition of Cones

Page 24

Close approximation of form by dividing into a number of pyramidal forms.
Volume calculation achieved by summation of simple formulae.



c. Durand's Rule

Page 26

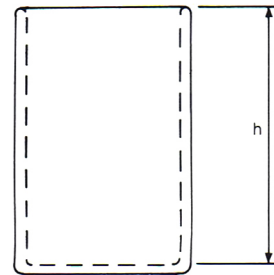
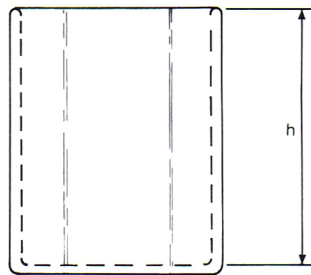
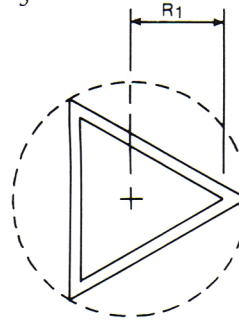
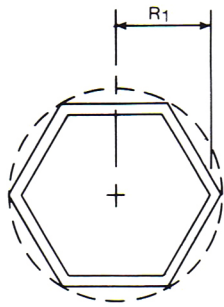
Dividing form into equal intervals and applying Durand's rule to obtain volume from dimensions of each interval.

Straight Sided Geometric Form (Polygonal)

Volume of Cylindrical Polygonal Shapes = $Dh R^2$

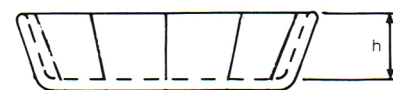
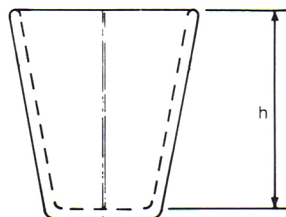
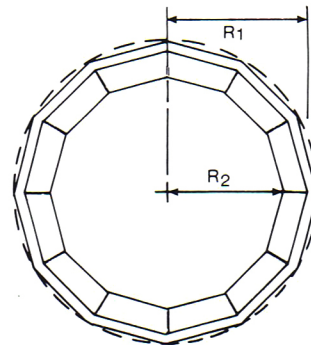
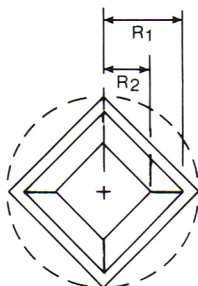
Volume of Truncated Pyramidal Shapes = $\frac{Dh}{3} (R_1^2 + R_2^2 + R_1 R_2)$

Volume of Truncated Pyramidal Shapes (Polygonal into Round) = $\frac{h}{3} [(2D - \pi) (R_1)^2 + D(R_2)^2 + DR_1R_2]$

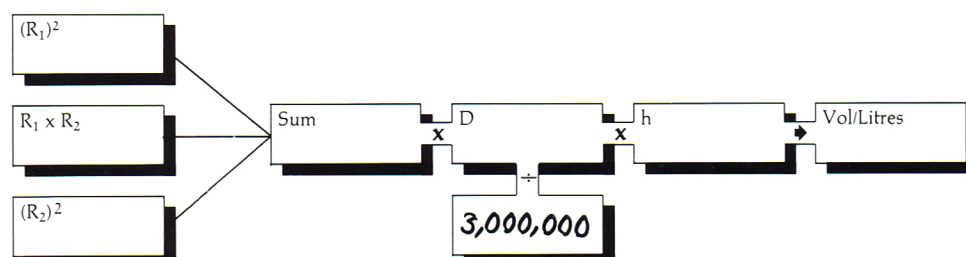


VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
13	3.0207
14	3.0372
15	3.0504
16	3.0615
17	3.0705
18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular

$R_1 = R_2$ for cylindrical shapes



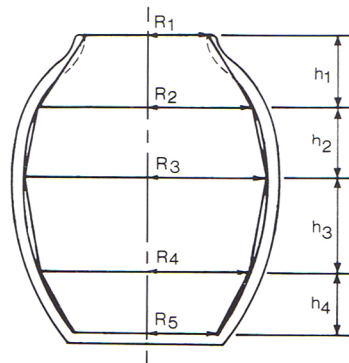
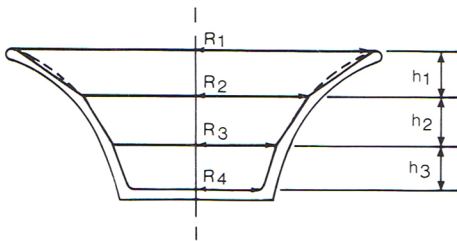
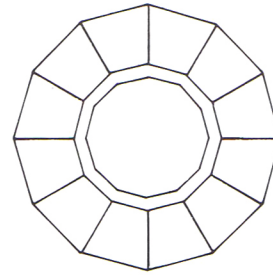
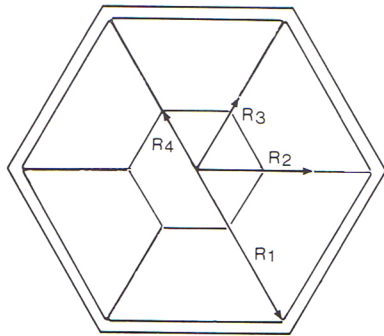
Radius of scribed circle around top polygon R_1 _____ mm
 Radius of scribed circle around bottom polygon R_2 _____ mm
 Constant D from look-up tables D _____
 Height of body h _____



Pyramidal Forms

$$\text{Volume} = \frac{D}{3} [h_1(R_1^2 + R_1 R_2 + R_2^2) + h_2(R_2^2 + R_2 R_3 + R_3^2) + \dots]$$

Note: The twisting of a polygonal form has no effect on its volume.



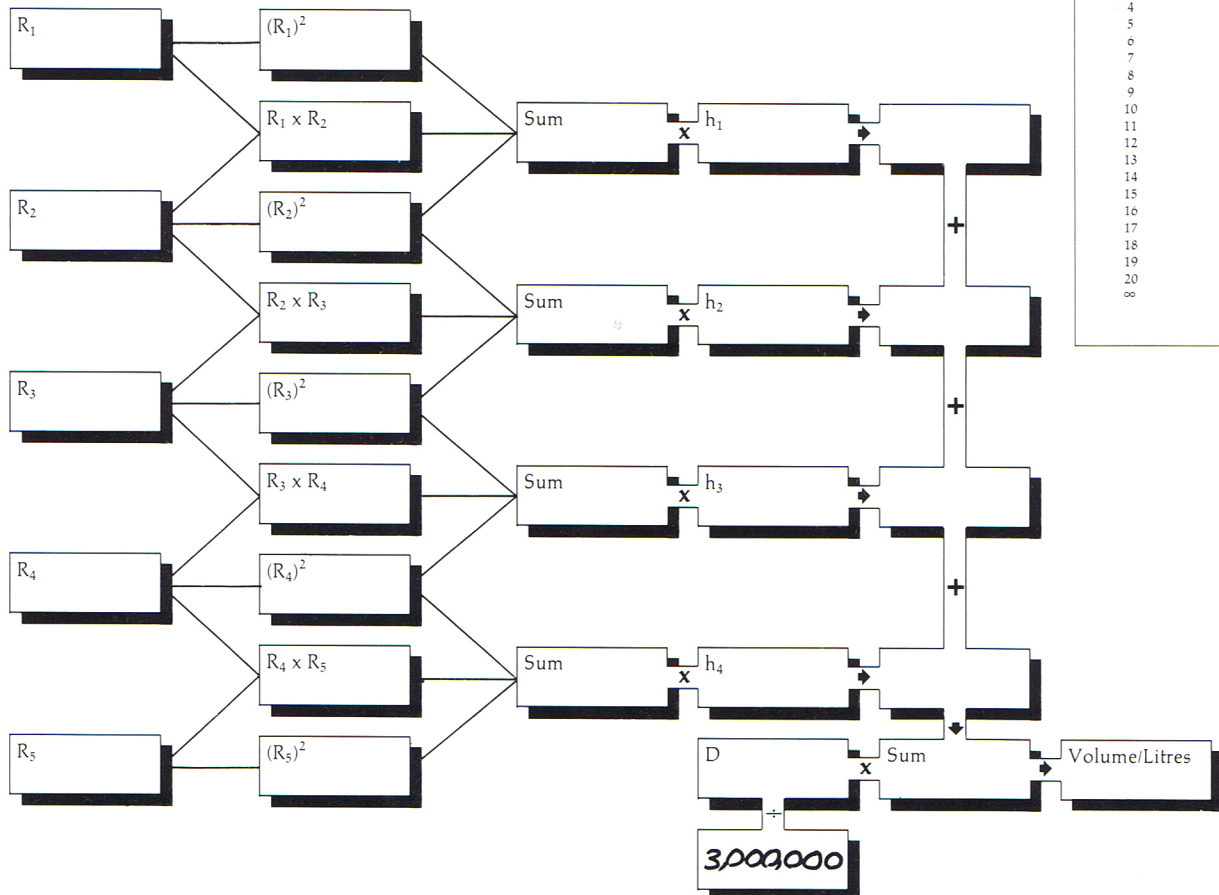
(The radius is always measured into the angle where the faces meet)
Note: the twisting of a polygonal form has no effect on its volume.

Interval radii $R_1 =$ $R_2 =$ $R_3 =$ $R_4 =$ $R_5 =$

Interval heights $h_1 =$ $h_2 =$ $h_3 =$ $h_4 =$

Number of facets $N =$

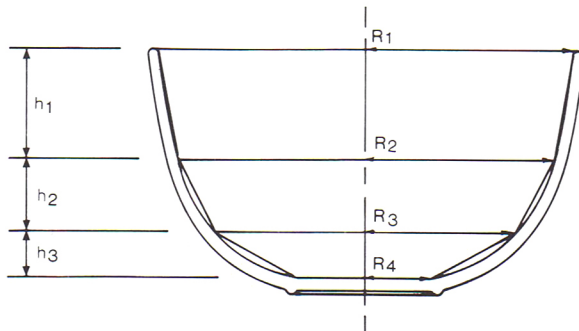
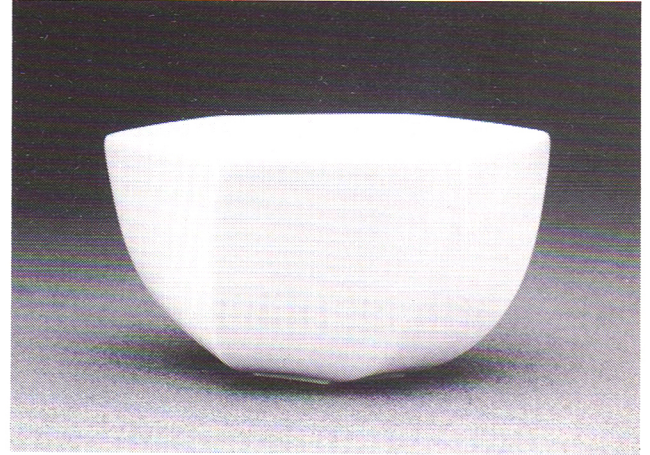
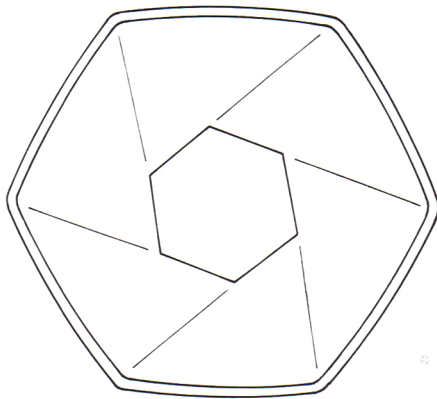
D is found from look-up tables $D =$



VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
13	3.0207
14	3.0372
15	3.0504
16	3.0615
17	3.0705
18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular

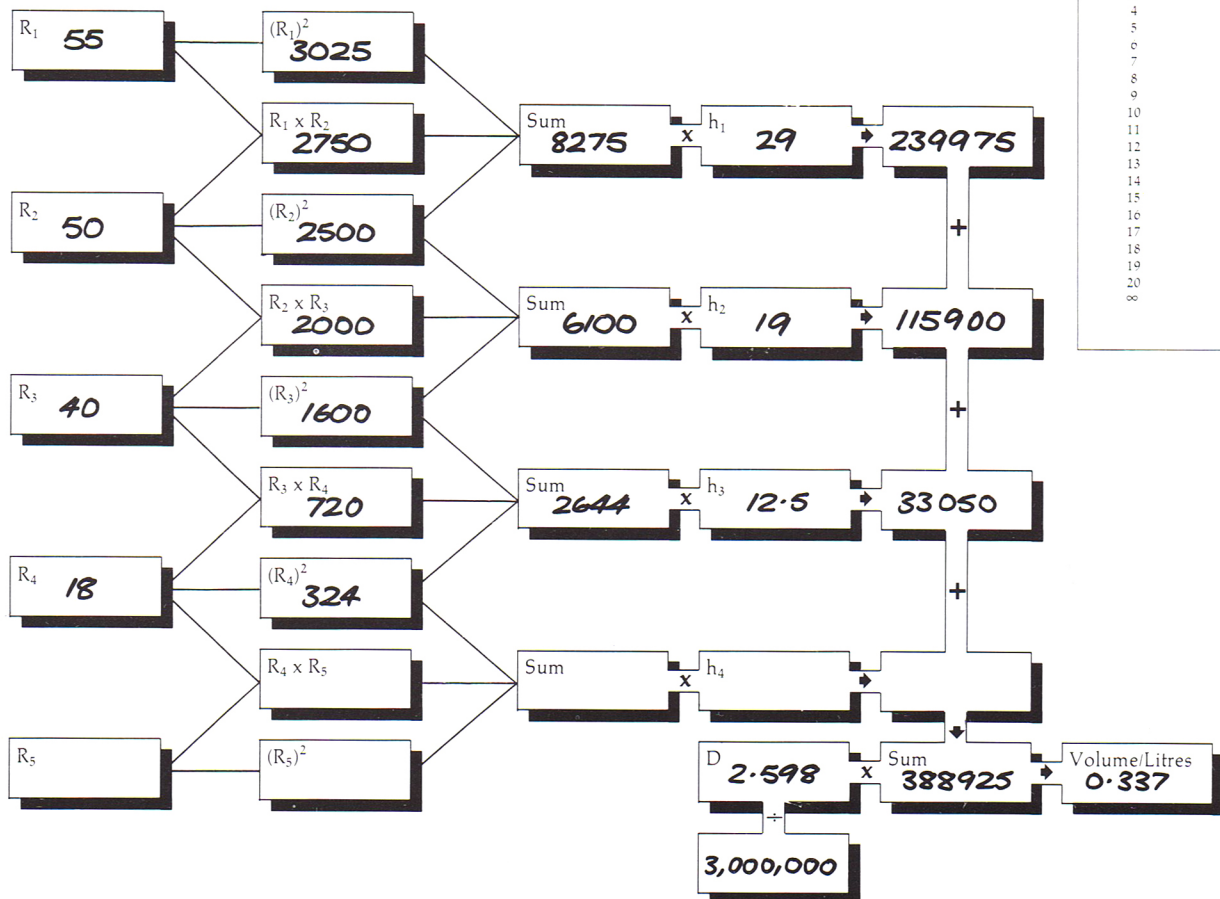
Example, Coalport Vase

Calculation by Cones (Polygonal)



Interval radii $R_1 = 55$ $R_2 = 50$ $R_3 = 40$ $R_4 = 18$ $R_5 =$
 Interval heights $h_1 = 29$ $h_2 = 19$ $h_3 = 12.5$ $h_4 =$
 Number of facets $N = 6$
 D is found from look-up tables $D = 2.598$

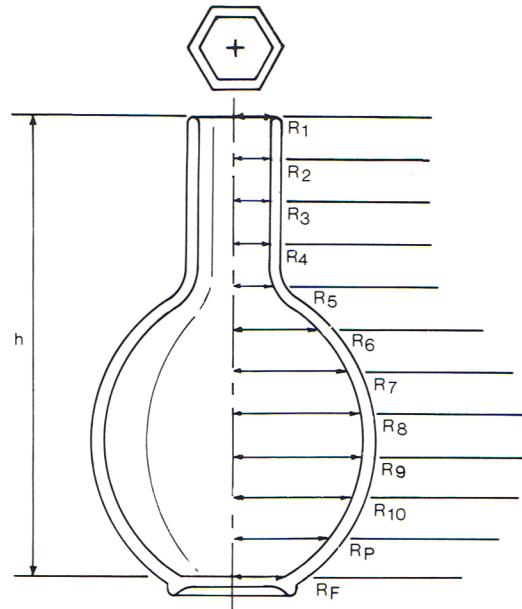
VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
13	3.0207
14	3.0372
15	3.0504
16	3.0615
17	3.0705
18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular



Durand's (Polygonal)

$$\text{Volume} = \frac{Dh}{N} [0.4(R_1)^2 + 1.1(R_2)^2 + (R_3)^2 + (R_4)^2 + \dots + 1.1(R_P)^2 + 0.4(R_F)^2]$$

Where R_P = Penultimate radius.
 R_F = Final radius.



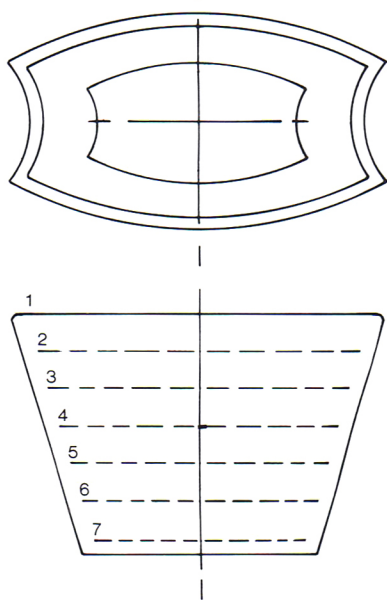
Radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____ $R_5 =$ _____
 $R_6 =$ _____ $R_7 =$ _____ $R_8 =$ _____ $R_9 =$ _____ $R_{10} =$ _____
 R (Penultimate) = _____ R (Final) = _____
 Overall height $h =$ _____ No of intervals $N =$ _____
 Constant D from look-up tables $D =$ _____

R_1	$(R_1)^2$	\times	0.4	\rightarrow	
R_2	$(R_2)^2$	\times	1.1	\rightarrow	
R_3	$(R_3)^2$	\rightarrow		\rightarrow	
R_4	$(R_4)^2$	\rightarrow		\rightarrow	
R_5	$(R_5)^2$	\rightarrow		\rightarrow	
R_6	$(R_6)^2$	\rightarrow		\rightarrow	
R_7	$(R_7)^2$	\rightarrow		\rightarrow	
R_8	$(R_8)^2$	\rightarrow		\rightarrow	
R_9	$(R_9)^2$	\rightarrow		\rightarrow	
R_{10}	$(R_{10})^2$	\rightarrow		\rightarrow	
R Penultimate	$(R \text{ Penultimate})^2$	\times	1.1	\rightarrow	
R Final	$(R \text{ Final})^2$	\times	0.4	\rightarrow	
h		\times	D	\rightarrow	Sum
N		\div		\rightarrow	Volume/Litres
			1,000,000		

VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
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16	3.0615
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18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular

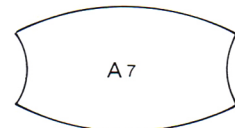
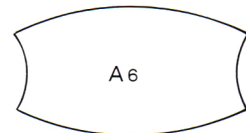
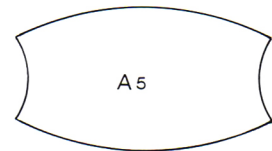
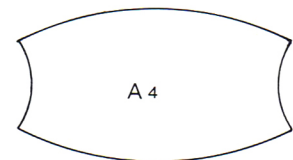
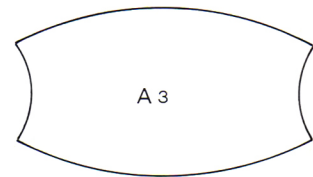
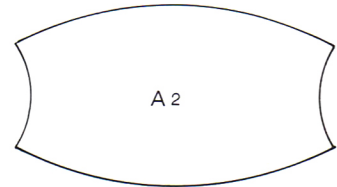
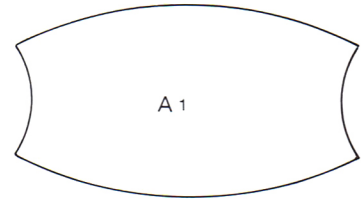
2.4 Irregular Polygons & Complex Forms

The volume of any form can be calculated using Durand's Rule, which is fundamentally a method of slicing the form into regular intervals and averaging the area measurements of all the cross-sections. The specific cases of using Durand's Rule for circular, elliptical and polygonal forms are covered in earlier chapters, but for irregular forms it is necessary to calculate the area of each cross-section.



Area $A_1 =$
 $A_2 =$
 $A_3 =$
 $A_4 =$
 $A_5 =$
 $A_6 =$
 $A_7 =$

The shape is divided into 6 equal intervals and the area of the 7 cross-sections calculated. The first and last cross-section represents the top and bottom shapes.



Three methods are described here for calculating areas:

1. Calculation

A shape can be simplified into triangles, rectangles and circles, then the component areas can be totalled together. More complicated shapes may require subtraction as well as addition to approximate the shape.

2. Graph Paper

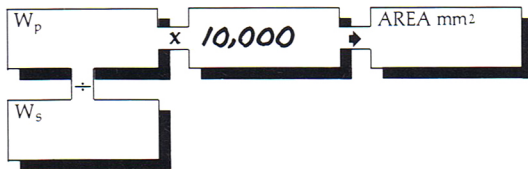
Each cross-section can be drawn on graph paper and by counting the squares within the shape the area can be directly determined.

3. Weighing

Each cross-section can be cut out of a sheet of card and then weighed individually. The weight of a square (measuring 100mm x 100mm) cut from the same card is also determined.

Weight of profile $W_p =$

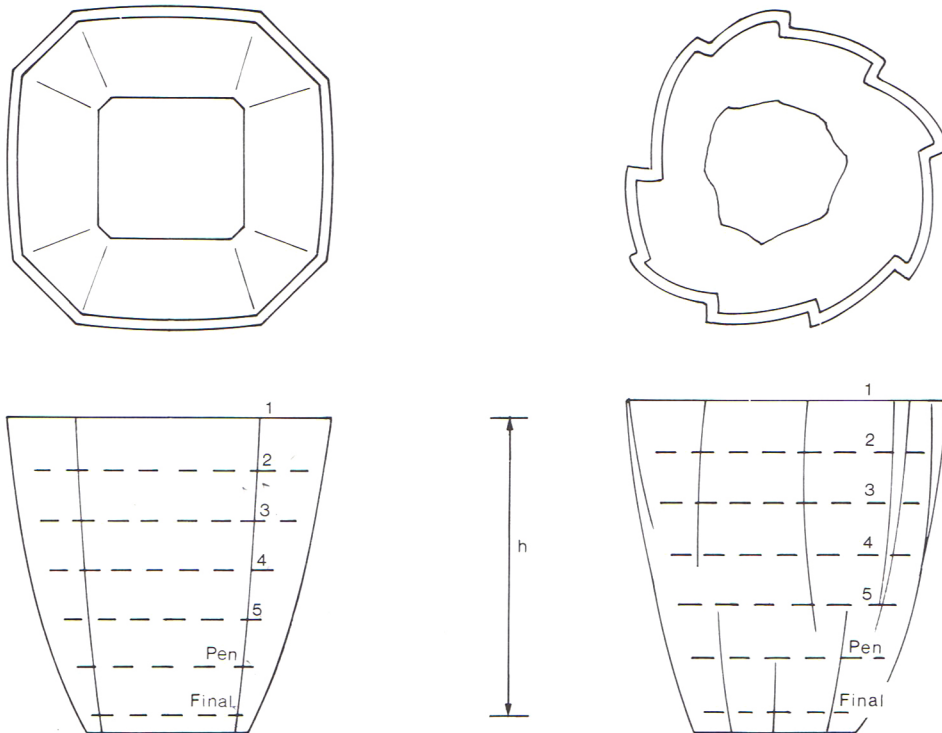
Weight of square $W_s =$



Durand's by Areas (Irregular Forms)

$$\text{Volume} = \frac{h}{N} [0.4A_1 + 1.1A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + 1.1A_p + 0.4A_f]$$

A_p = Penultimate area.
 A_f = Final area.



Note: Lateral displacement or twisting of the constituent sections alters the form of the object but has no effect on its volume. This process is known as shearing

Number of intervals $N =$ _____

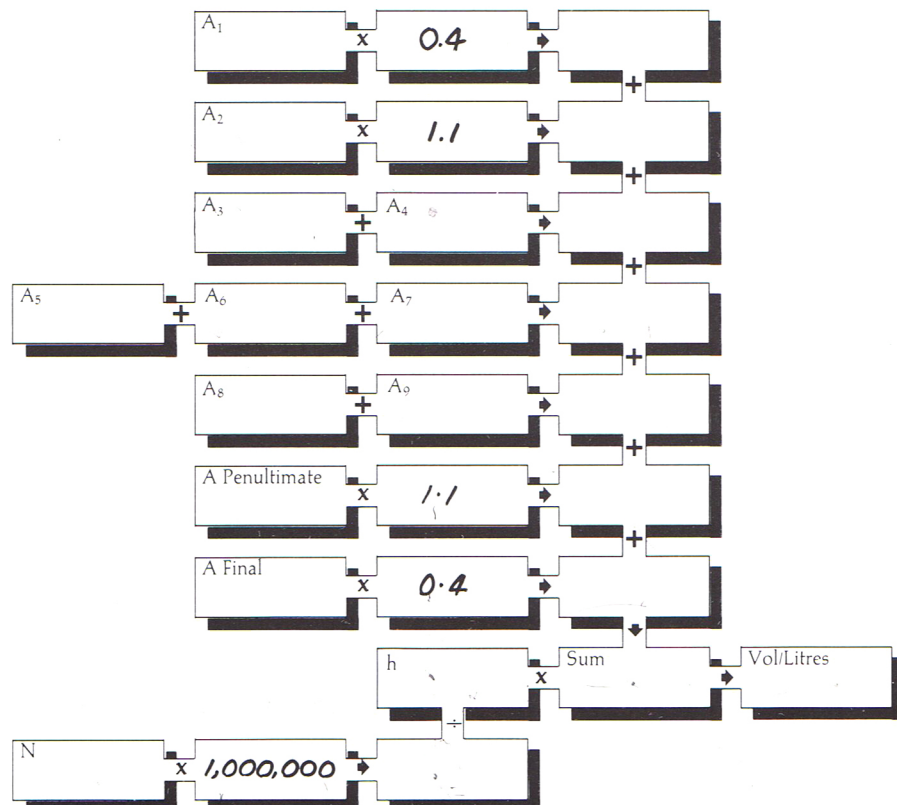
Height $h =$ _____ mm

Areas A_1 _____ mm² A_2 _____ mm² A_3 _____ mm²

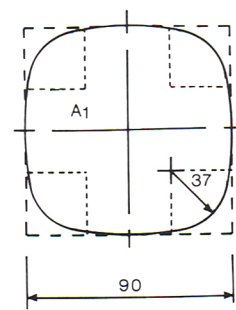
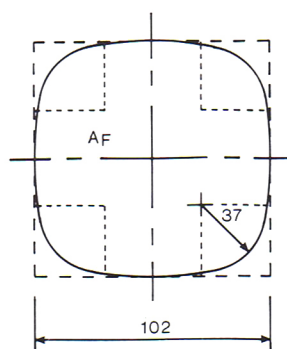
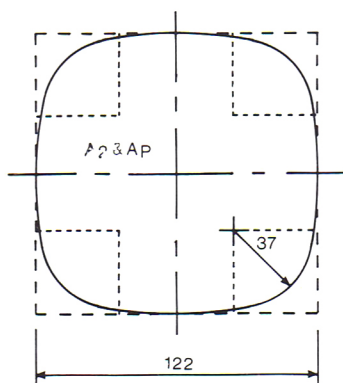
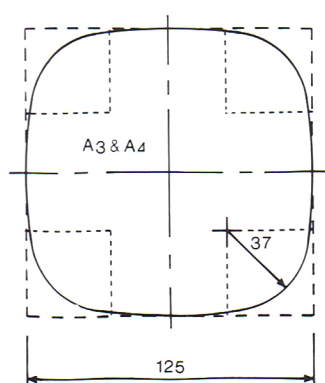
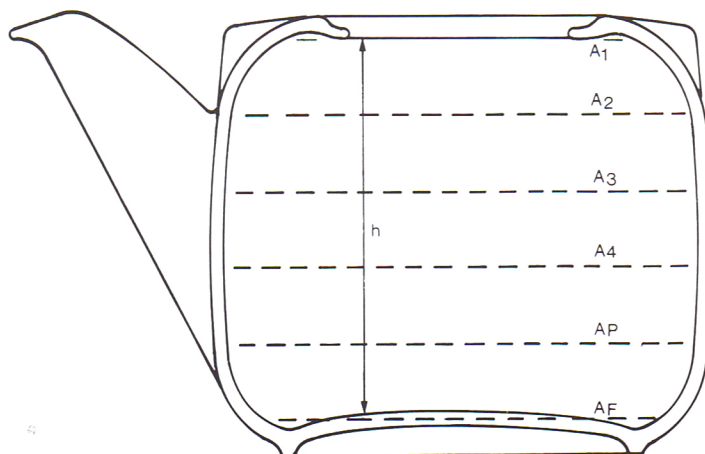
A_4 _____ mm² A_5 _____ mm² A_6 _____ mm²

A_7 _____ mm² A_8 _____ mm² A_9 _____ mm²

$A(\text{Pen})$ _____ mm² $A(\text{Final})$ _____ mm²

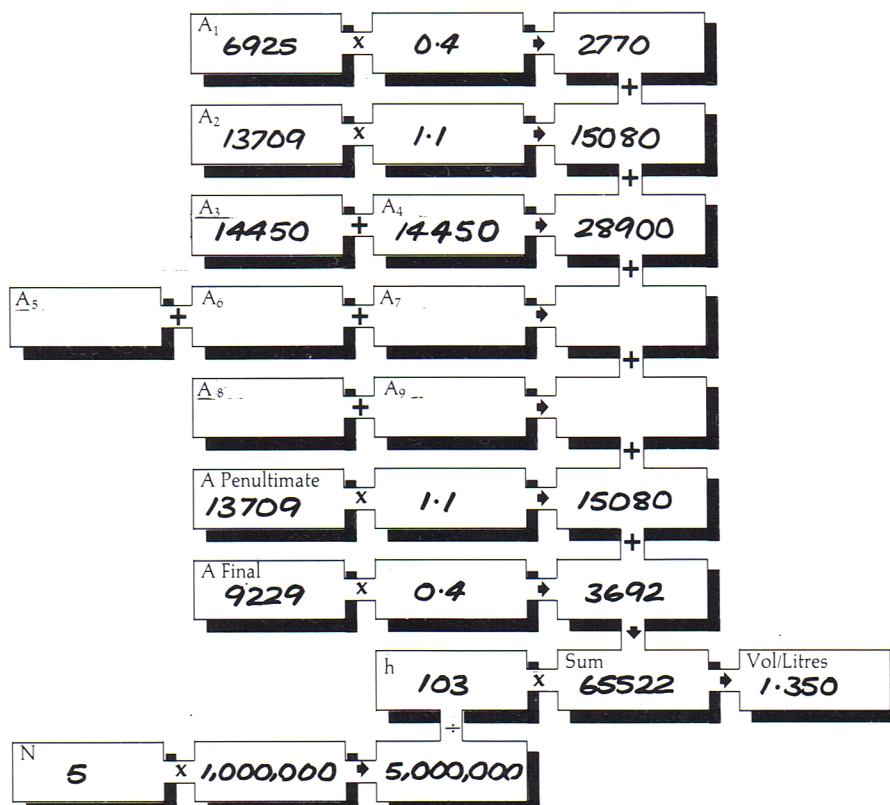


Example, Suomi Teapot



Number of intervals $N = 5$
 Height $h = 103$ mm
 Areas A_1 6925 mm² A_2 13709 mm² A_3 14450 mm²
 A_4 14450 mm² A_5 _____ mm² A_6 _____ mm²
 A_7 _____ mm² A_8 _____ mm² A_9 _____ mm²
 A (Pen) 13709 mm² A (Final) 9229 mm²

The above diagrams show a method of calculating areas by a simplification of the geometry. The Suomi teapot is sliced and each section drawn up (two repeats). As the shape can be approximated into a rounded square the area calculation was a subtraction of the corner fillet areas from the overall square. The workings of these calculations are not shown here.

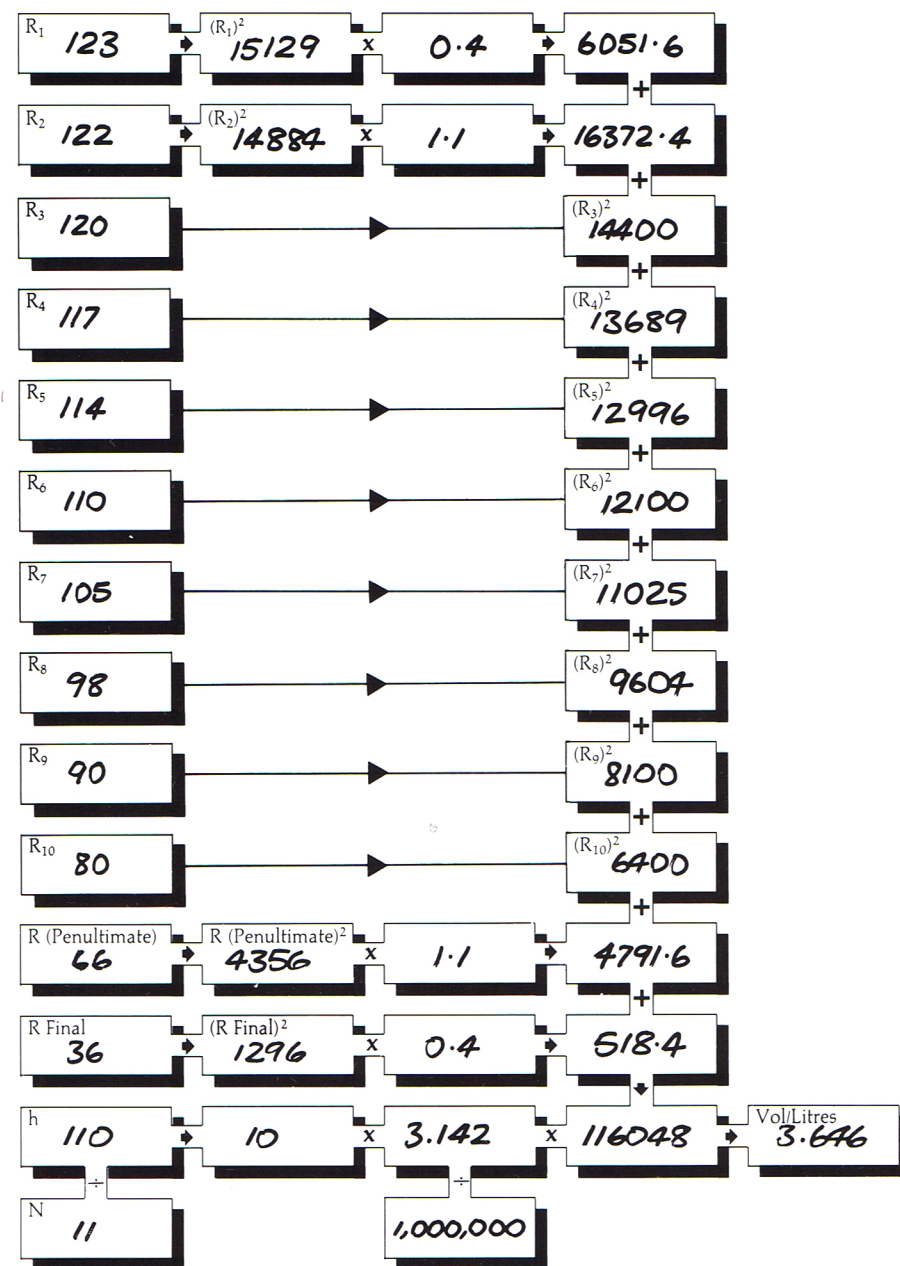


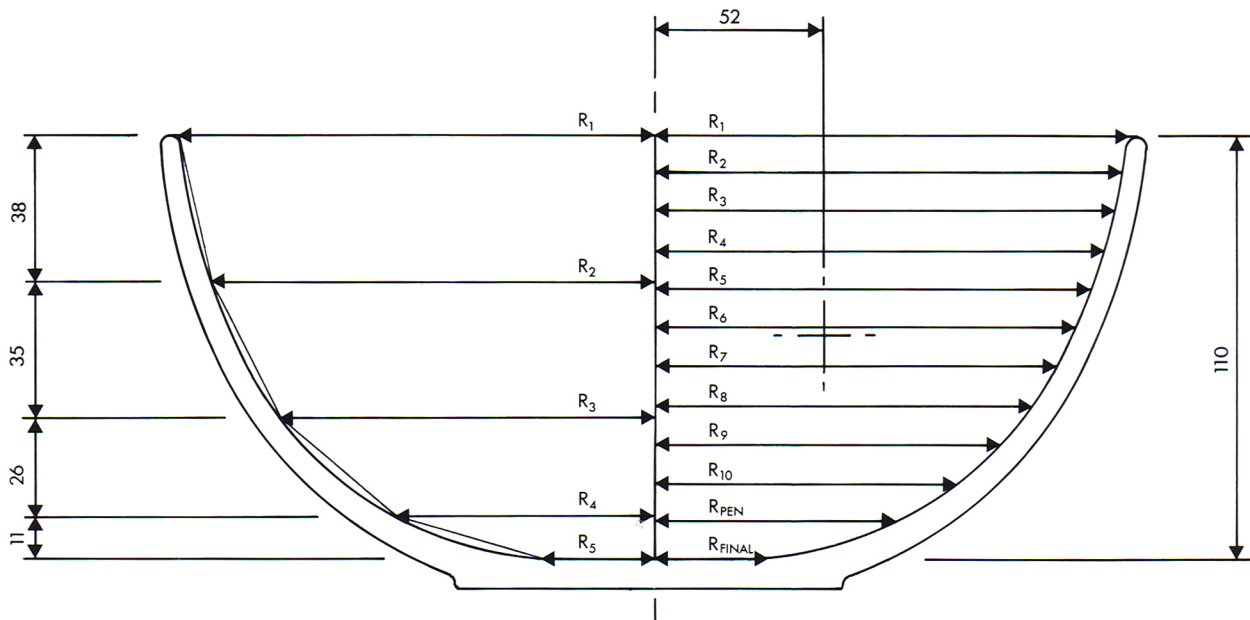
3. Comparison of Calculation Methods

All measurements are worked throughout in millimetres. The resultant is divided by 1,000,000 converting cubic millimetres (millilitres) into litres.

1. Durand's (Circular Form)

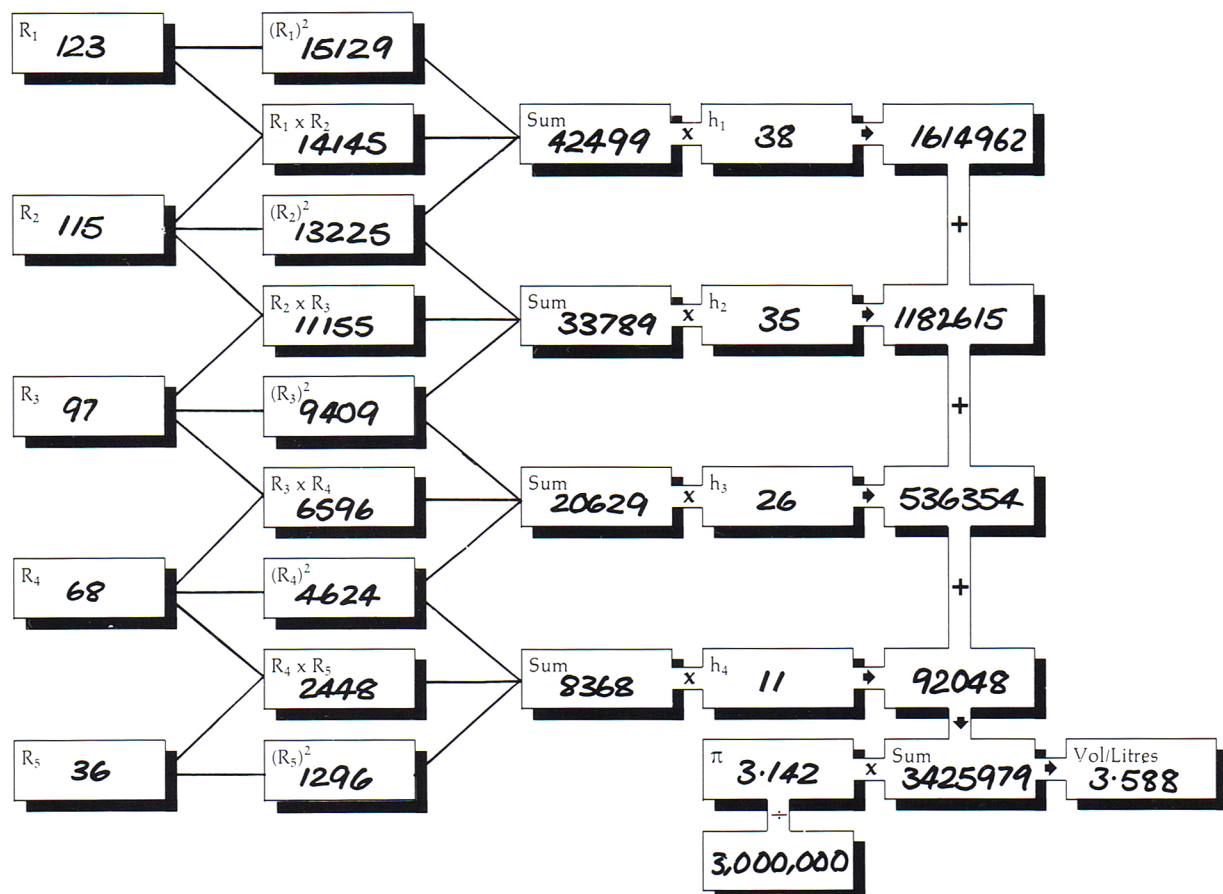
Radii R_1 123 R_2 122 R_3 120 R_4 117 R_5 114
 R_6 110 R_7 105 R_8 98 R_9 90 R_{10} 80
 R (Penultimate) = 66 R (Final) = 36
 Overall height h = 110 No of intervals N = 11





2. Addition of Cones (Circular Form)

Slice radii R_1 123 R_2 115 R_3 97 R_4 68 R_5 36
 Slice heights h_1 38 h_2 35 h_3 26 h_4 11



3. Averaging Radii

R₁ 123
R₂ 122
R₃ 120
R₄ 117
R₅ 114
R₆ 110
R₇ 105
R₈ 98
R₉ 90
R₁₀ 80
R₁₁ 66
R₁₂ 36

1181

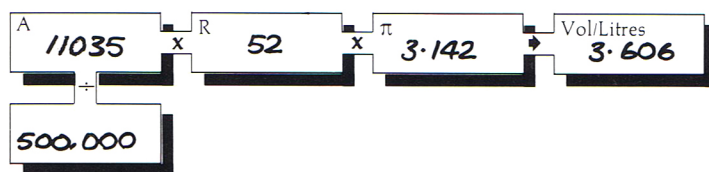
$$\text{VOLUME} = 3.142 \times 98.42 \times 98.42 \times 110$$

$$= 3.348 \text{ litre}$$

$$\text{Average radii} = \frac{1181}{12} = 98.42\text{mm}$$

4. Pappus' Theorem

Area by calculation A = 11035mm²
Radius to C of G R = 52mm



Summary

1. Durand's	3.646 litre
2. Cones	3.588 litre
3. Averaging radii	3.348 litre
4. Pappus' Theorem	3.606 litre

Conclusion

Calculation of the volume by Durand's method, the Cones method and by using Pappus' theorem all give reasonable results. These three results vary by less than 1.6%. Calculation by the first method is expected to give an accurate result. The second method yield a lower volume; this is a result of the cones all being described completely within the shape, leaving thin pieces at the periphery (between the straight lines and the curve) ignored in the calculation. Pappus' theorem will also yield an accurate answer (errors are introduced by inaccuracies in measuring the area and determining the centre of gravity).

The third method, averaging the radii, give an erroneous result. This method is mathematically incorrect, and will, in most circumstances, produce a wrong answer (more extreme curves give far worse results). It is unfortunate that this method has been the most frequently taught to students at colleges, and it is recommended that this method should never be used. The first method, Durand's rule, will give an accurate answer and involves no more calculations than averaging the radii.

In terms of usefulness, Durand's method is accurate but involves making many measurements, the Cones method is almost as accurate and involves much fewer measurements. Pappus' theorem is very practical and involves making only two measurements but the accuracy is very much dependant on the skill of the user.

4. Scaling Three Dimensional Volumetric Scaling

Volume Increase %	Linear Increase %	Multiplication Factor
1	0.3	1.0033
2	0.7	1.0066
3	1.0	1.0099
4	1.3	1.0132
5	1.6	1.0164
6	2.0	1.0196
7	2.3	1.0228
8	2.6	1.0260
9	2.9	1.0291
10	3.2	1.0323
11	3.5	1.0354
12	3.8	1.0385
13	4.2	1.0416
14	4.5	1.0446
15	4.8	1.0477
16	5.1	1.0507
17	5.4	1.0537
18	5.7	1.0567
19	6.0	1.0597
20	6.3	1.0627
21	6.6	1.0656
22	6.9	1.0685
23	7.1	1.0714
24	7.4	1.0743
25	7.7	1.0772
26	8.0	1.0801
27	8.3	1.0829
28	8.6	1.0858
29	8.9	1.0886
30	9.1	1.0914
31	9.4	1.0942
32	9.7	1.0970
33	10.0	1.0997
34	10.2	1.1025
35	10.5	1.1052
36	10.8	1.1079
37	11.1	1.1106
38	11.3	1.1133
39	11.6	1.1160
40	11.9	1.1187
41	12.1	1.1213
42	12.4	1.1240
43	12.7	1.1266
44	12.9	1.1292
45	13.2	1.1319
46	13.4	1.1344
47	13.7	1.1370
48	14.0	1.1396
49	14.2	1.1422
50	14.5	1.1447

Volume Decrease %	Linear Decrease %	Multiplication Factor
1	0.3	0.9967
2	0.7	0.9933
3	1.0	0.9899
4	1.3	0.9865
5	1.7	0.9830
6	2.0	0.9796
7	2.4	0.9761
8	2.7	0.9726
9	3.1	0.9691
10	3.4	0.9655
11	3.8	0.9619
12	4.2	0.9583
13	4.5	0.9546
14	4.9	0.9510
15	5.3	0.9473
16	5.7	0.9435
17	6.0	0.9398
18	6.4	0.9360
19	6.8	0.9322
20	7.2	0.9283
21	7.6	0.9244
22	8.0	0.9205
23	8.3	0.9166
24	8.7	0.9126
25	9.1	0.9086
26	9.6	0.9045
27	10.0	0.9004
28	10.4	0.8963
29	10.8	0.8921
30	11.2	0.8879
31	11.6	0.8837
32	12.1	0.8794
33	12.5	0.8750
34	12.9	0.8707
35	13.4	0.8662
36	13.8	0.8618
37	14.3	0.8573
38	14.7	0.8527
39	15.2	0.8481
40	15.7	0.8434
41	16.1	0.8387
42	16.6	0.8340
43	17.1	0.8291
44	17.6	0.8243
45	18.1	0.8193
46	18.6	0.8143
47	19.1	0.8093
48	19.6	0.8041
49	20.1	0.7990
50	20.6	0.7937

It is often necessary to change the capacity of a design by increasing or decreasing its dimensions. For example, if a designed pot has a volume of say 0.8 litres and it is desired to increase this by 25% to 1 litre, then it is necessary to know by what percentage its linear dimensions (usually length, breadth and height) be increased.

Using the three-dimensional scaling table, it can be seen that a 25% volume increase would necessitate a 7.7% linear increase (or multiply all linear measurements by 1.0772).

This table gives values for volume changes from a 50% decrease to a 50% increase. If the required volume change is outside this range, then the calculation should be split into two or more stages.

e.g. what linear change is required to decrease the volume of an object from 2 litre down to 0.6 litres?

2 litre less 50% = 1 litre (multiplication factor = 0.7937)

1 litre less 40% = 0.6 litre (multiplication factor = 0.8434)

Overall linear multiplication factor = 0.7937×0.8434
= 0.6694

Two Dimensional Volumetric Scaling Proportional

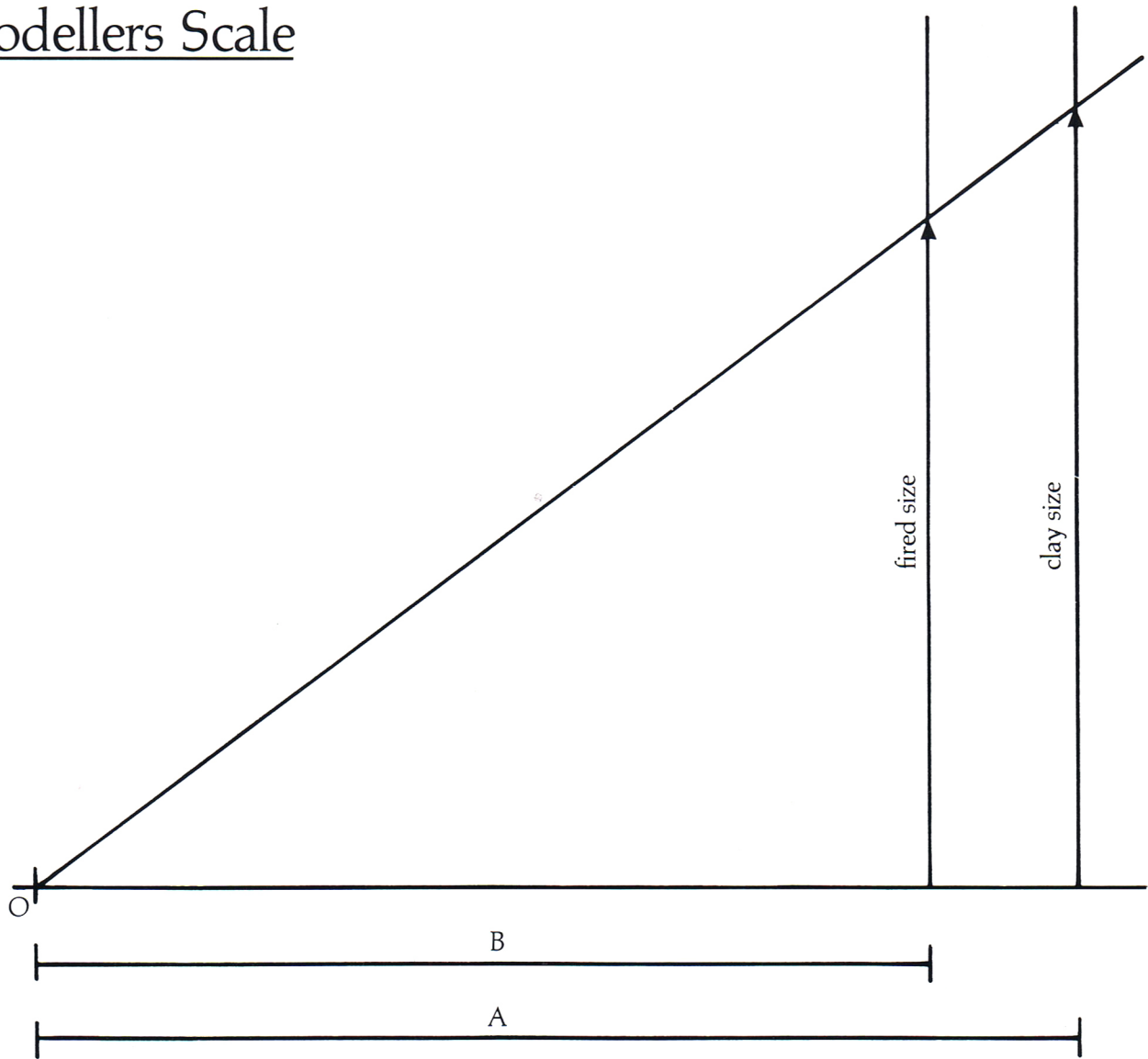
Volume Increase %	Linear Increase %	Multiplication Factor
1	0.5	1.0050
2	1.0	1.0100
3	1.5	1.0149
4	2.0	1.0198
5	2.5	1.0247
6	3.0	1.0296
7	3.4	1.0344
8	3.9	1.0392
9	4.4	1.0440
10	4.9	1.0488
11	5.4	1.0536
12	5.8	1.0583
13	6.3	1.0630
14	6.8	1.0677
15	7.2	1.0724
16	7.7	1.0770
17	8.2	1.0817
18	8.6	1.0863
19	9.1	1.0909
20	9.5	1.0954
21	10.0	1.1000
22	10.5	1.1045
23	10.9	1.1091
24	11.4	1.1136
25	11.8	1.1180
26	12.3	1.1225
27	12.7	1.1269
28	13.1	1.1314
29	13.6	1.1358
30	14.0	1.1402
31	14.5	1.1446
32	14.9	1.1489
33	15.3	1.1533
34	15.8	1.1576
35	16.2	1.1619
36	16.6	1.1662
37	17.1	1.1705
38	17.5	1.1747
39	17.9	1.1790
40	18.3	1.1832
41	18.7	1.1874
42	19.2	1.1916
43	19.6	1.1958
44	20.0	1.2000
45	20.4	1.2042
46	20.8	1.2083
47	21.2	1.2124
48	21.7	1.2166
49	22.1	1.2207
50	22.5	1.2247

Volume Decrease %	Linear Decrease %	Multiplication Factor
1	0.5	0.9950
2	1.0	0.9899
3	1.5	0.9849
4	2.0	0.9798
5	2.5	0.9747
6	3.0	0.9695
7	3.6	0.9644
8	4.1	0.9592
9	4.6	0.9539
10	5.1	0.9487
11	5.7	0.9434
12	6.2	0.9381
13	6.7	0.9327
14	7.3	0.9274
15	7.8	0.9220
16	8.4	0.9165
17	8.9	0.9110
18	9.5	0.9055
19	10.0	0.9000
20	10.6	0.8944
21	11.1	0.8888
22	11.7	0.8832
23	12.3	0.8775
24	12.8	0.8718
25	13.4	0.8660
26	14.0	0.8602
27	14.6	0.8544
28	15.2	0.8485
29	15.7	0.8426
30	16.3	0.8367
31	16.9	0.8307
32	17.5	0.8246
33	18.2	0.8185
34	18.8	0.8124
35	19.4	0.8062
36	20.0	0.8000
37	20.6	0.7937
38	21.3	0.7874
39	21.9	0.7810
40	22.5	0.7746
41	23.2	0.7681
42	23.8	0.7616
43	24.5	0.7550
44	25.2	0.7483
45	25.8	0.7416
46	26.5	0.7348
47	27.2	0.7280
48	27.9	0.7211
49	28.6	0.7141
50	29.3	0.7071

Sometimes it is desirable to change only the linear measurements in two dimensions, keeping the third dimension fixed. For example, if it was desired to increase the capacity of a 0.8 litre pot by 25% with the restriction that the height should not change, then it is necessary to know by how much the horizontal measurements should be increased. Using the two-dimensional scaling table, it can be seen that an 11.8% increase (or multiply by 1.1180) in the horizontal measurements (length and breadth) is required.

If the required volume change is beyond the limits of the table then the calculation should be split into two or more stages.

Modellers Scale



A modellers scale is a practical method of scaling up or down a model or a drawing. It is widely used in the ceramic industry by modellers who have to take into account the shrinkage that occurs in most ceramic bodies and can be used for any form of proportional scaling. For example:

Earthenware has a shrinkage of 1 in 12 so a plate made from a 12" diameter mould will give an 11" diameter fired plate. To make a modellers scale for earthenware shrinkage proceed as follows:

Draw a base line from point 0 of any length but for convenience choose a length that is divisible by 12 (say 24 cms), then strike a point on your base line that is $\frac{1}{12}$ shorter, ie. 22 cms. Erect 2 perpendiculars from the base line at 22 and 24 cms from point 0. Any measurement made on one of the perpendiculars if drawn through in a straight line to point 0 will cross the other perpendicular, giving an 11 to 12 relationship. This scale is widely used by modellers using calipers working from a fired size to a clay size model; they do not need to know the actual measurements that they are taking.

5. Thicknesses that might be expected with different materials and products

It is only possible to calculate the volume of a vessel from a drawing or model if you know the wall thickness of the material from which the article is to be made. The following list gives the approximate wall thicknesses to be expected with holloware items made in ceramic, glass and certain other materials.

CERAMIC TABLEWARE

<u>Materials</u>	<u>mm</u>
Cup Bone China	2.5 – 3
Cream, Bone China	3
Sugar Bone China	3
Coffeepot Bone China	3.5 – 4
Teapot Bone China	3.5 – 4
Gravy Boat Bone China	4
Mug Bone China	2.5 – 3
Cast Giftware Bone China	3
Cup Earthenware	3.5 – 4
Cream Earthenware	4
Sugar Earthenware	4 – 5
Coffeepot Earthenware	4
Teapot Earthenware	4
Mug Earthenware	4 – 5
Cookware Earthenware	5 – 7
Cookware Porcelain	5 – 7
Cookware Stoneware	5 – 7

Tableware Porcelain high quality as bone china

Tableware Porcelain low quality as earthenware

Tableware Stoneware as earthenware

GLASS HOLLOWARE

Stemware Handblown	not cut	1 – 2
Stemware Machine made	high quality	1.5 – 2
Stemware Cut Crystal	hand blown	2 – 2.5
Tumblers Machine made	high quality	2 – 3
Tumblers Hand Blown	not cut	2 – 3
Tumblers Cut Crystal	hand blown	2 – 3
Cookware Pressed	small pieces	4 – 5
Cookware Pressed	large pieces	5 – 6
Storage Jars	machine made	4 – 6
Jugs, Decanters	hand blown not cut	3 – 5
Bottles	machine made	2.5

METAL COOKWARE

Cast iron	4 – 6
Enamel Steel	1.5 – 2
Aluminium	2.4
Stainless Steel	1

Useful Capacities for Ceramic Tableware

Perceived capacity is what really matters and the following must be seen only as a general guide:

		<u>litres</u>
Breakfast Cup	most markets	.35
Tea Cup	most markets	.25
Coffee Cup	Germany/Scandinavia	.20
Espresso Cup	Italy/Greece	.10
Sugar	most markets	.35
Cream	most markets	.35
Cream	Germany	.18
Gravy Boat	most markets	.50
Tea Pot	most markets	1.20
Coffee Pot	most markets	1.20
Vegetable Casserole Dish	most markets	1.75

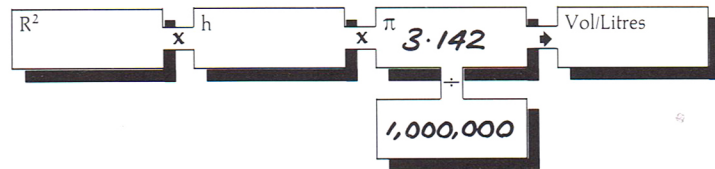
6. Calculation sheets for photocopying

All measurements must be worked in millimetres. The resultant is divided converting cubic millimetres (millilitres) into litres.

Cylindrical Forms (Circular) see page 8

Radius R = _____

Height h = _____

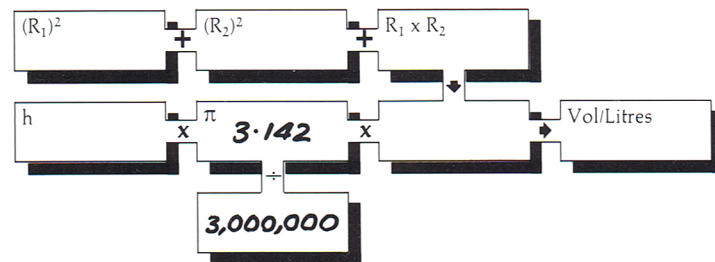


Truncated Cones (Circular) see page 9

Top Radius R_1 = _____

Height h = _____

Bottom Radius R_2 = _____

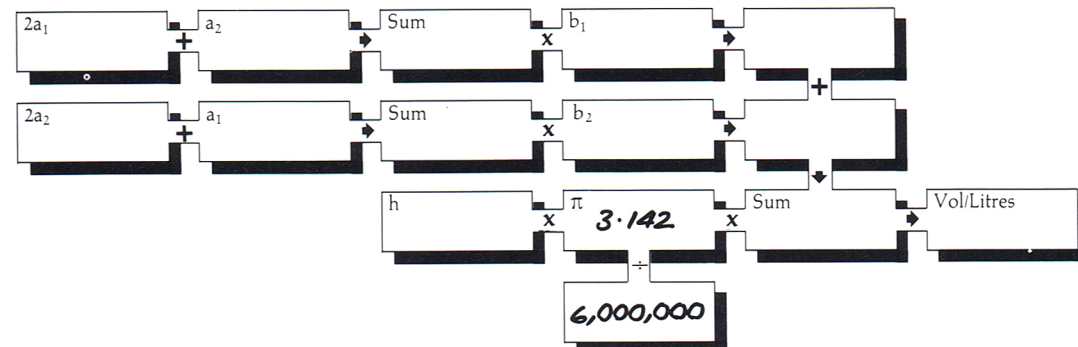


Straight Sided Elliptical Forms see page 17

Major axis radii a_1 = _____ a_2 = _____

Minor axis radii b_1 = _____ b_2 = _____

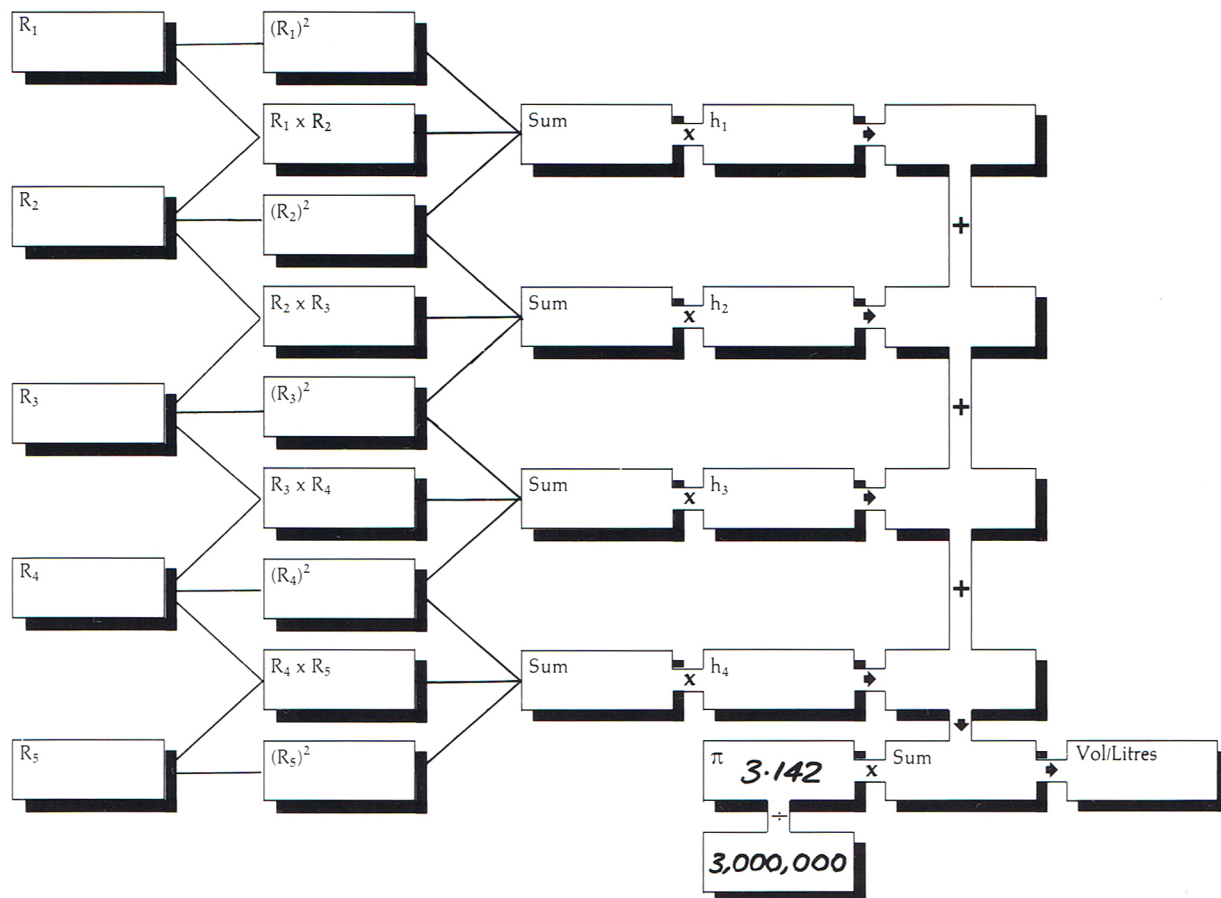
Height h = _____



Addition of Cones (Circular Forms) see page 10

Internal radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____ $R_5 =$ _____

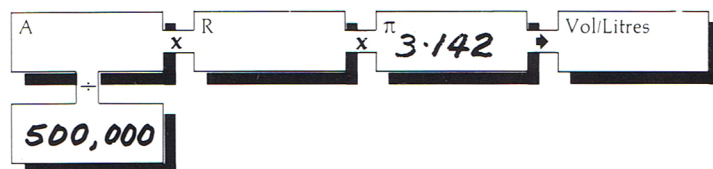
$h_1 =$ _____ $h_2 =$ _____ $h_3 =$ _____ $h_4 =$ _____



Pappus' Theorem (Circular Forms) see page 14

Area of half profile A = _____ mm²

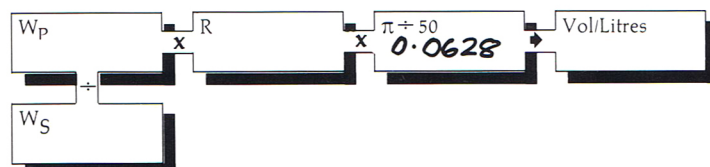
Radius of C of G R = _____ mm



Weight of profile (grammes) $W_p =$ _____ g

Weight of square (grammes) $W_s =$ _____ g

Radius to C & G (millimetres) R = _____ mm



All measurements must be worked in millimetres.
The resultant is divided converting cubic millimetres (millilitres) into litres.

Durand's Rule (Circular Forms) see page 12

Radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____ $R_5 =$ _____

$R_6 =$ _____ $R_7 =$ _____ $R_8 =$ _____ $R_9 =$ _____ $R_{10} =$ _____

R (Penultimate) = _____

R (Final) = _____

Overall height h = _____

No. of intervals N = _____

R_1	$(R_1)^2$	x	0.4	+	
R_2	$(R_2)^2$	x	1.1	+	
R_3	$(R_3)^2$			+	
R_4	$(R_4)^2$			+	
R_5	$(R_5)^2$			+	
R_6	$(R_6)^2$			+	
R_7	$(R_7)^2$			+	
R_8	$(R_8)^2$			+	
R_9	$(R_9)^2$			+	
R_{10}	$(R_{10})^2$			+	
R Penultimate	$(R \text{ Penultimate})^2$	x	1.1	+	
R Final	$(R \text{ Final})^2$	x	0.4	+	
h		x	π 3.142	x	Sum
N			1,000,000		Vol/Litres

All measurements must be worked in millimetres. The resultant is divided converting cubic millimetres (millilitres) into litres.

Addition of Cones (Elliptical Forms) see page 18

Height $h_1 =$ _____ $h_2 =$ _____ $h_3 =$ _____

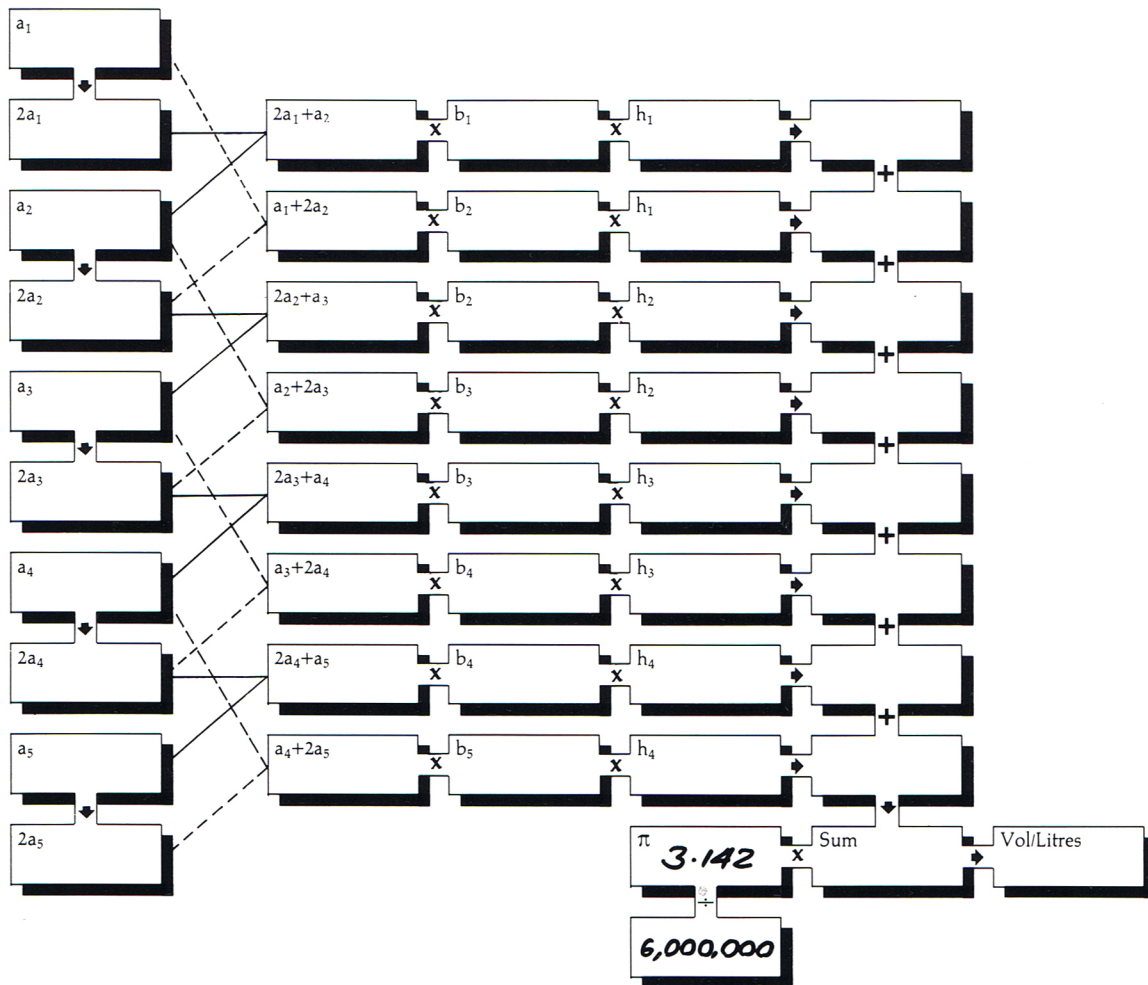
$h_4 =$ _____

$a_1 =$ _____ $a_2 =$ _____ $a_3 =$ _____

$a_4 =$ _____ $a_5 =$ _____

$b_1 =$ _____ $b_2 =$ _____ $b_3 =$ _____

$b_4 =$ _____ $b_5 =$ _____



All measurements must be worked in millimetres. The resultant is divided converting cubic millimetres (millilitres) into litres.

Durand's Rule (Elliptical Forms) see page 20

Major axis $a_1 =$ _____ $a_2 =$ _____ $a_3 =$ _____ $a_4 =$ _____

$a_5 =$ _____ $a_6 =$ _____ $a_7 =$ _____ $a_8 =$ _____

$a_9 =$ _____ $a_{10} =$ _____ $a =$ _____ $a =$ _____
Penultimate Final

Minor axis $b_1 =$ _____ $b_2 =$ _____ $b_3 =$ _____ $b_4 =$ _____

$b_5 =$ _____ $b_6 =$ _____ $b_7 =$ _____ $b_8 =$ _____

$b_9 =$ _____ $b_{10} =$ _____ $b =$ _____ $b =$ _____
Penultimate Final

Height $h =$ _____ No. of Intervals $N =$ _____

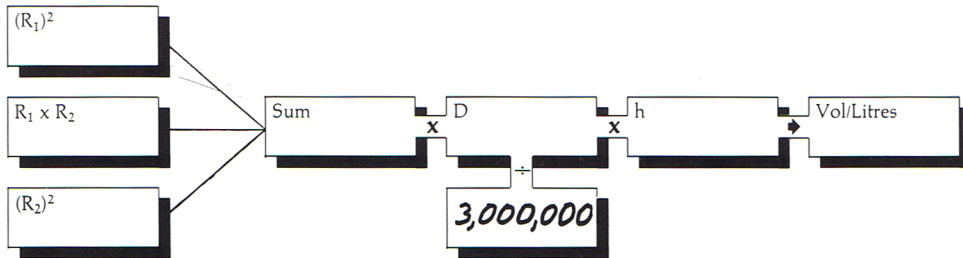
a_1	x	b_1	x	0.4	→	
a_2	x	b_2	x	1.1	→	
a_3	x	b_3			→	
a_4	x	b_4			→	
a_5	x	b_5			→	
a_6	x	b_6			→	
a_7	x	b_7			→	
a_8	x	b_8			→	
a_9	x	b_9			→	
a_{10}	x	b_{10}			→	
a Penultimate	x	b Penultimate	x	1.1	→	
a Final	x	b Final	x	0.4	→	
h	+		x	π 3.142	x	Sum
N				1,000,000		Vol/Litres

All measurements must be worked in millimetres.

The resultant is divided by 1,000,000 converting cubic millimetres (millilitres) into litres.

Straight Sided Geometric Forms (Polygonal) see page 23

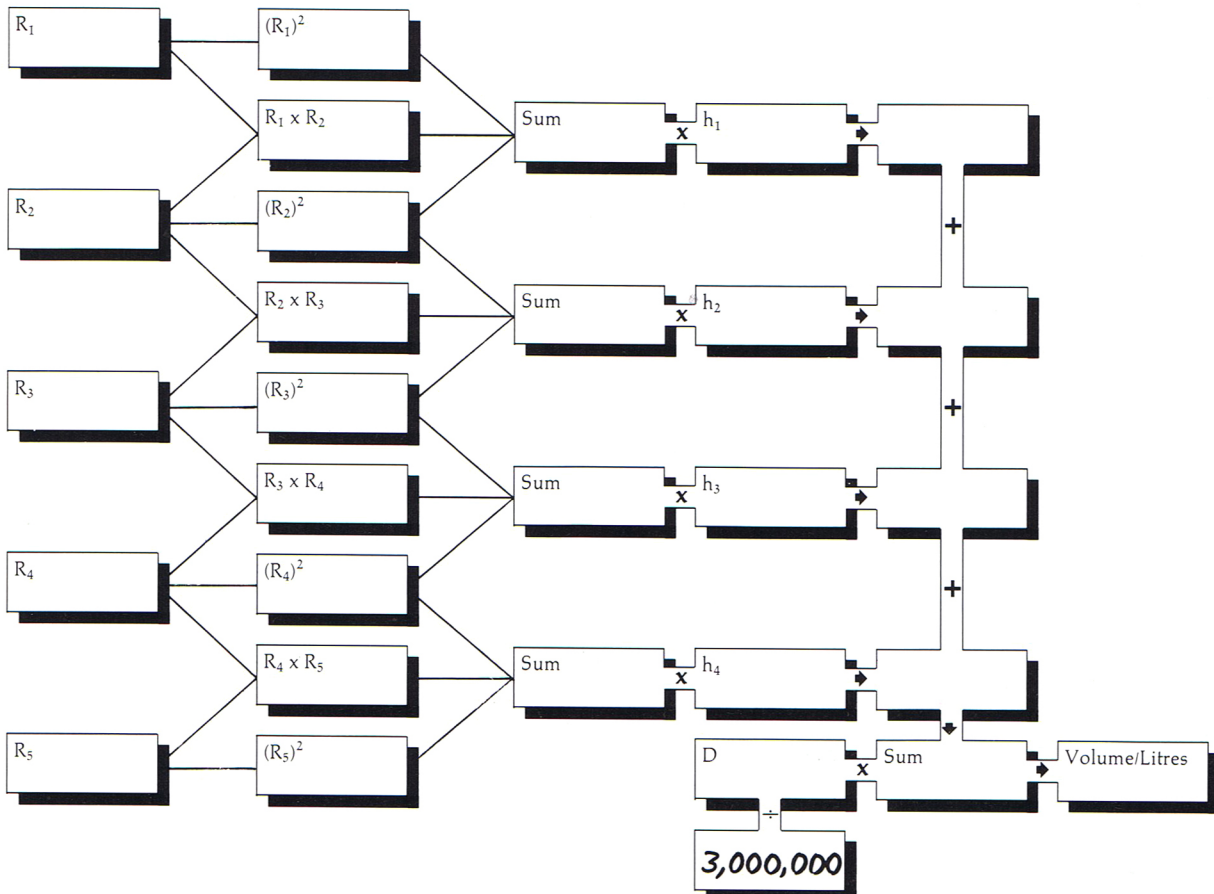
Radius of scribed circle around top polygon R_1 _____ All measurements must be
 Radius of scribed circle around bottom polygon R_2 _____ worked in millimetres.
 Constant D from look-up tables D _____ The resultant is divided
 Height of body h _____ converting cubic millimetres
 (millilitres) into litres.
 $R_1 = R_2$ for cylindrical shapes



VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
13	3.0207
14	3.0372
15	3.0504
16	3.0615
17	3.0705
18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular

Pyramidal Forms see page 24

Interval heights $h_1 =$ _____ $h_2 =$ _____ $h_3 =$ _____ $h_4 =$ _____
 Interval radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____
 Number of facets $N =$ _____ $R_5 =$ _____
 D is found from look-up table $D =$ _____



Durand's Rule (Polygonal Forms) see page 26

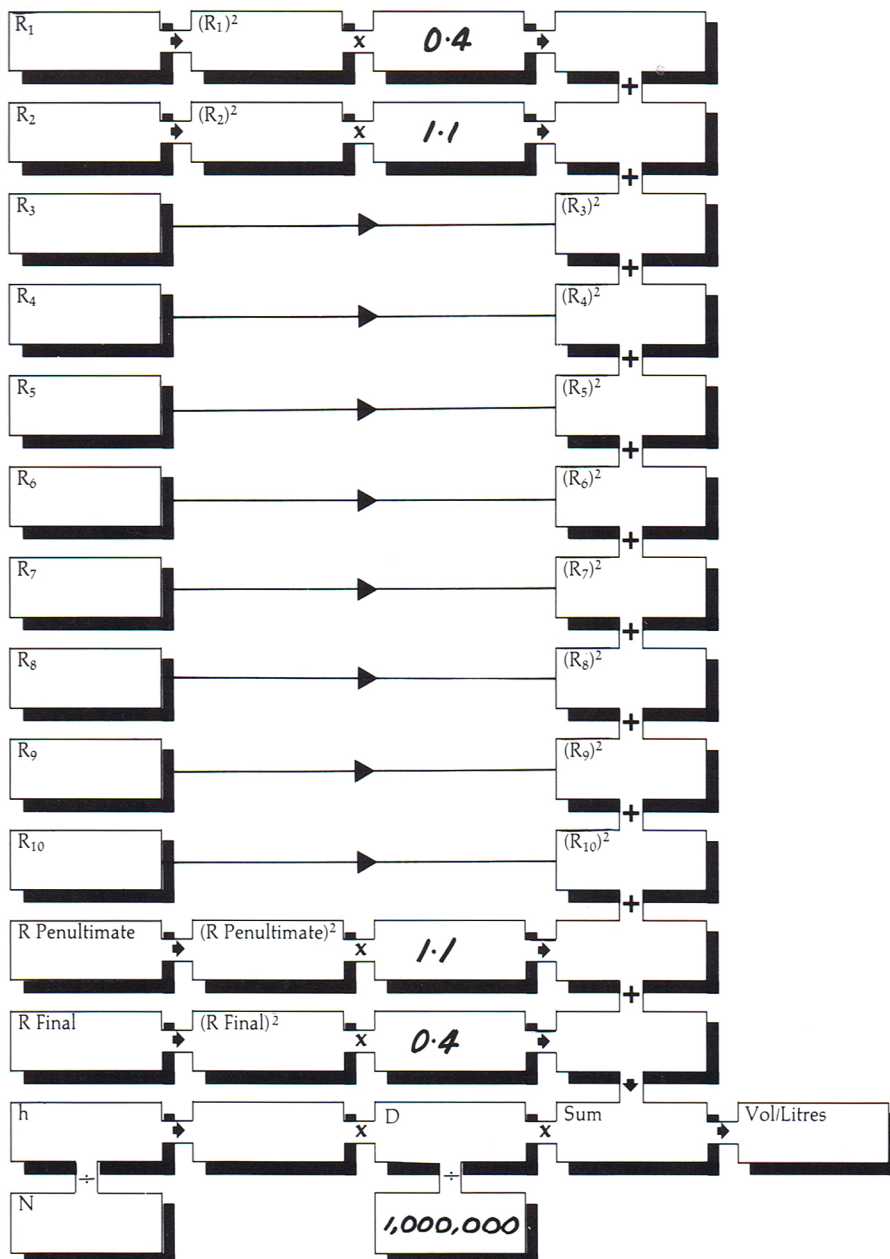
Radii $R_1 =$ _____ $R_2 =$ _____ $R_3 =$ _____ $R_4 =$ _____ $R_5 =$ _____

$R_6 =$ _____ $R_7 =$ _____ $R_8 =$ _____ $R_9 =$ _____ $R_{10} =$ _____

R (Penultimate) = _____ R (Final) = _____

Overall height $h =$ _____ No of intervals $N =$ _____

Constant D from look-up tables $D =$ _____



VALUE OF D	
No of sides of Polygon	
3	1.299
4	2.0
5	2.3775
6	2.598
7	2.7363
8	2.8284
9	2.8926
10	2.9388
11	2.9736
12	3.0
13	3.0207
14	3.0372
15	3.0504
16	3.0615
17	3.0705
18	3.0783
19	3.0846
20	3.0903
∞	3.142 (π) ie. circular

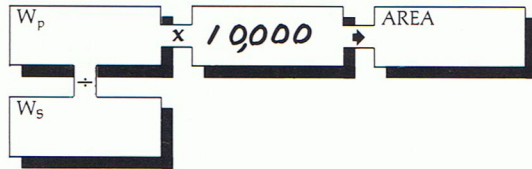
All measurements must be worked in millimetres. The resultant is divided by 1,000,000 converting cubic millimetres (millilitres) into litres.

Durand's Rule

Irregular Polygonal & Complex Forms see page 28

Weight of profile $W_p =$ _____ g

Weight of square $W_s =$ _____ g



All measurements must be worked in millimetres. The resultant is divided by 1,000,000 converting cubic millimetres (millilitres) into litres.

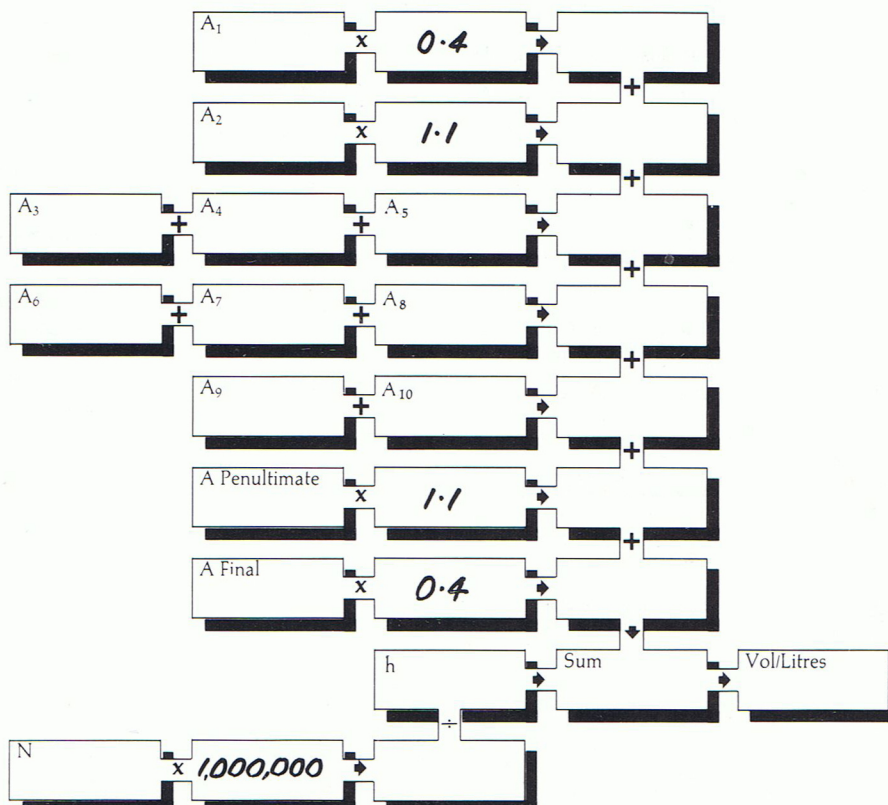
Height $h =$ _____ mm Number of intervals $N =$ _____

Areas A_1 _____ mm^2 A_2 _____ mm^2 A_3 _____ mm^2

A_4 _____ mm^2 A_5 _____ mm^2 A_6 _____ mm^2

A_7 _____ mm^2 A_8 _____ mm^2 A_9 _____ mm^2

A (Pen) _____ mm^2 A (Final) _____ mm^2



7. Factors, Densities & Formulae

Conversion Factors (Metric/Imperial)

To change	into	multiply by	To change	into	multiply by
LENGTH					
Inches	Centimetres	2.5400	Centimetres	Inches	0.3937
AREA					
Square inches	Square centimetres	6.4516	Square centimetres	Square inches	0.1550
VOLUME					
Cubic inches	Cubic centimetres	16.387	Cubic centimetres	Cubic inches	0.06102
Cubic feet	Litres	28.317	Litres	Cubic feet	0.03531
	Cubic metres	0.02832	Cubic metres		35.311
Cubic feet	Imperial gallons	6.237	Imperial gallons	Cubic feet	0.1603
Cubic yards	Cubic metres	0.7645	Cubic metres	Cubic yards	1.3080
Imperial gallons	Litres	4.5460	Litres	Imperial gallons	0.2200
Imperial gallons	U.S. gallons	1.205	U.S. gallons	Imperial gallons	0.830
Pints	Litres	0.5682	Litres	Pints	1.7598
MASS					
Grains	Grams	0.0648	Grams	Grains	15.432
Ounces (avoir)	Grams	28.352	Grams	Ounces (avoir)	0.03527
Pounds	Kilograms	0.4536	Kilograms	Pounds	2.20462

Densities of Ceramic Materials

Earthenware	2.2 g/cm ³
Stoneware	2.3 g/cm ³
Porcelain	2.4 g/cm ³ - 2.7 g/cm ³
Bone China	2.5 g/cm ³ - 2.8 g/cm ³

Volumes of Circular Forms

Cylindrical (see page 8)

$$V = \pi R^2 h$$

Truncated Cones (page 9)

$$V = \frac{\pi}{3} h (R_1^2 + R_2^2 + R_1 R_2)$$

Addition of Cones (page 10)

$$V = \frac{\pi}{3} [(R_1^2 + R_2^2 + R_1 R_2) h_1 + (R_2^2 + R_3^2 + R_2 R_3) h_2 + \dots]$$

Durand's Rule (page 12)

$$V = \pi h \left(0.4 R_1^2 + 1.1 R_2^2 + R_3^2 + R_4^2 + \dots + 1.1 R_p^2 + 0.4 R_F^2 \right)$$

Pappus' Theorem (page 14)

$$V = 2 \pi A R$$

Volumes of Elliptical Forms

Cylindrical & Truncated Forms (page 17)

$$V = \frac{\pi}{6} h ((2a_1 + a_2) b_1 + (a_1 + 2a_2) b_2)$$

Addition of Cones (page 18)

$$V = \frac{\pi}{6} ((2a_1 + a_2) b_1 h_1 + (a_1 + 2a_2) b_2 h_1 + (2a_2 + a_3) b_2 h_2 + (a_2 + 2a_3) b_3 h_2 + \dots)$$

Durand's Rule (Page 20)

$$V = \frac{h}{N} \pi (0.4a_1 b_1 + 1.1a_2 b_2 + a_3 b_3 + a_4 b_4 + \dots + 1.1a_{Pen} b_{Pen} + 0.4a_{Final} b_{Final})$$

Irregular Forms

Using Durand's Method. (Page 28)

$$V = \frac{h}{N} \pi (0.4A_1 + 1.1A_2 + A_3 + A_4 + \dots + 1.1A_{Pen} + 0.4A_{Final})$$

Volumes of Polygons

Cylindrical Shapes (page 23)

$$V = D h R^2$$

Truncated Pyramids (page 23)

$$V = \frac{Dh}{3} (R_1^2 + R_2^2 + R_1 R_2)$$

Addition of Cones (page 24)

$$V = \frac{D}{3} ((R_1^2 + R_2^2 + R_1 R_2) h_1 + (R_2^2 + R_3^2 + R_2 R_3) h_2 + \dots)$$

Durand's Method. (page 26)

$$V = Dh \left(0.4R_1^2 + 1.1R_2^2 + R_3^2 + R_4^2 + \dots + 1.1R_{Pen}^2 + 0.4R_{Final}^2 \right)$$

Area Equations

Rectangle $A = xy$

Circle $A = \pi R^2$

Eclipse $A = \pi ab$

Polygon $A = DR^2$

Volume Equations

Sphere $V = \frac{4}{3} \pi r^3$

Cone $V = \frac{1}{3} \pi R^2 h$

QUEENSBERRY HUNT

Queensberry Hunt is one of England's leading product design groups. They are particularly well known for their work in the ceramic field. The partners have between them won six Design Council Awards and the German Bundes Preis (Gute Form). The partner's work is also well represented in the permanent collection of the Victoria & Albert Museum. David Queensberry, who was Professor of Ceramics and Glass at the Royal College of Art until 1984, founded the group with one of his students, Martin Hunt RDI, in 1966. In recent years two younger designers have become partners, Robin Levien and John Horler. The partnership has worked for many distinguished manufacturers and retailers, these include:

American Standard
British Telecom
Corning
Dartington Crystal
Habitat
Hornsea Pottery
Ideal Standard
Marks & Spencer

Mikasa
Pilkington Glass
Pilkington Tiles
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Rosenthal China
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Siam Fine China
Thomas China

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Pottery crafts Limited is the country's leading manufacturer and supplier of pottery materials and equipment for craft, hobby, education and small industrial use. The company has one of the largest ranges in the world covering the whole spectrum of potters' needs from clays and glazes to kilns, potters wheels, tools and books.

Pottery crafts is not simply a source of supply, the company also has a commitment to providing technical help and advice and furthering the advance of knowledge and skill in all aspects of hand made ceramics.